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ON THE USE

OF THE

AUTOMATIC PHOTOMETER

IN PHOTOGRAPHIC PHOTOMETRY

A THESIS

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I hereby declare that the work involved in the accompanying thesis (except where otherwise distinctly stated) has been performed entirely by me, and that it is entirely composed by me.

Royal Observatory,
Edinburgh.

Dec. 21st, 1926.

C O N T E N T S

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A P P E N D I C E S

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1. THE LAW OF BLACKENING OF THE PHOTOGRAPHIC PLATE AT LOW DENSITIES

   I. Plan of the Investigation

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   III. Bibliography of Work on the Validity of the Reciprocity Law

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   V. A Theory of the Deviations from the Reciprocity Law.

   VI. The Influence of Wavelength on Density Growth

   (Parts I - V are from the Proceedings of the Royal Society of Edinburgh, Vols. xlv & xlvii.)

2. ON THE VALIDITY OF TALBOT'S LAW FOR THE PHOTOGRAPHIC PLATE

   (From the Proceedings of the Optical Convention, 1926, Part I.)

3. A CONVENIENT PHOTO-ELECTRIC PHOTOMETER AND DENSITOMETER

   (From the Journal of Scientific Instruments, Vol. I.)

4. DESCRIPTION OF THE KOCH MICRO-PHOTOMETER OF THE ROYAL OBSERVATORY, EDINBURGH

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INTRODUCTION

The displacement of the eye of the astronomer by the photographic plate from the telescope and spectrometer and its transfer to the measuring machine was so rapid that about twenty years ago practical astronomy seemed likely to become a branch of applied photography. More recently the introduction of the photo-electric cell and improvements in the thermopile and selenium cell have extended this process of displacement, not from the telescope only, but also from the microscope of the measuring machine. The sensational shift of the lines in the companion of Sirius, for instance, was measured by a thermo-electric device. One of the first to advocate the newer methods of measuring a photographic plate was the late E. C. Pickering*; his appeal for an automatic means of measuring was answered shortly after by the introduction first of the Koch and later of other instruments. Fortunately or unfortunately, the manufacture of the photographic plate has become so specialised that advances in technique must be left in commercial hands, it remains for the astronomer to see that the fullest use is made of its capabilities, and while the element of personal judgment in the measurement of the negative remains this duty cannot be said to have been performed.

It is a natural tendency for the user of an electrical mesuring device to study the properties of the

* *Harvard Circular*, 155, 1920.

photographic plate rather than to devote much attention to the improvement of the measuring device, for the reason that the accuracy is limited by the defects of the plate and the methods of development, the errors of measurement being negligible. The automatic instrument, moreover, throws into prominence the weaker parts of the images which passed unnoticed by the eye. It is a peculiar feature of the eye, and one that leads to considerable error in some instances, that it is incapable of perceiving gradual changes in illumination. An abrupt change of illumination of one per cent can be seen, a gradual change of one hundred per cent may pass unnoticed; hence as a criterion of uniformity of illumination the eye is not merely useless, it is misleading. The use of low densities is an obvious advantage in astronomy, for they require less exposure than dense images, and an extension to fainter stars becomes possible. The idea of measuring the positions and magnitudes of star images which are invisible on the negative may appear strange, it is nevertheless true that a star image of mean density less than 0.03 will not be picked up by eye, whereas a suitable photometer will locate and measure images down to a mean density of 0.01. The behaviour of the photographic plate in this very low density region is of interest, not only for its astronomical applications, but also because of the simplicity of the results and the light they throw on the photographic action.

The thesis consists of three parts. The first, which comprises chapters I to V, treats of the errors to be expected in automatic measures of stellar images arising from the properties of the photographic plate and from its treatment. The second, comprising chapters VI to VIII, is an account of an attempt to measure the photo-visual magnitudes of the brighter stars of the North Polar Sequence as an example of the use of the photometer for image measurement, together with the considerations which led to the adoption of the method used. The third comprises four appendices, containing that part of the work which is not of exclusively astronomical interest. The first of these is to be regarded as part of the main thesis - it constitutes, in fact, the basis of the whole work.

This thesis could not have been prepared without Professor Sampson's consent; but the writer is indebted to him for much more than this would imply, and where the apparatus used is of an improvised or inadequate type this is in no way due to his lack of sympathy with the work - on the contrary, the writer has to thank him for the consideration he has given to requests, whether for apparatus or for advice on points of difficulty. The writer has also to thank Sir James Walker for facilities for preparing photo-electric cells in the Chemical Department of the University of Edinburgh.

I

THE PHOTOGRAPHIC PLATE

The gelatine dry plate consists of a layer of gelatine about 30 μ thick coated on a glass plate and containing minute crystals of silver bromide and iodide, together with an active constituent containing sulphur, the exact nature of which is still under discussion. In fast emulsions the diameters of the grains range from ultra-microscopic dimensions up to about 4μ; in process emulsions the diameters of the largest grains do not rise above 1μ. The control exercised by the manufacturers over the speed, colour sensitiveness and certain other properties of the emulsion, at any rate as regards fast colour sensitive plates, is far from complete, so that it is dangerous to come to general conclusions from the study of a few emulsions. They take considerable pains, however, to ensure that the plates belonging to a single batch are as homogeneous as possible; and this homogeneity constitutes for photographic photometry one of the most valuable features of the plate. In view of the complexities of the actions taking place in photography it has sometimes been urged that in photometry the plate should be used solely as a test of the equality of two beams of light.* Ultimately all methods of photographic photometry reduce to this; but by taking advantage of the uniformity of a batch of plates it is frequently possible to use more efficient methods of working than strict

* *J. Baillaud, Bull. Astron. iv, 1923, p. 286; F. C. Toy, Nature, cxvii, 1926, p. 83.*

equalising permits.

Previous to its development the emulsion is slightly translucent, it also reflects diffusely a considerable proportion of the incident light. Both reflected and transmitted light are weak in violet and ultra-violet compared with the light source, giving the emulsion a yellowish-green colour; the same remarks apply to the light scattered from directly illuminated areas into neighbouring parts of the emulsion. Isochromatic and panchromatic emulsions contain dyes which render the plate more opaque to the yellow and orange respectively. Undyed or blue-sensitive plates are practically insensitive to light of wavelength longer than 5,000 Å; isochromatic plates are sensitive up to nearly 6,000 Å; while panchromatic plates lose the greater part of their sensitiveness at 6,600 Å and in stellar work may be regarded as quite insensitive beyond 7,000 Å. If the energy falling on unit area is taken as measuring the illumination it appears that the plate is most sensitive in the violet and that relatively the sensitiveness of the best panchromatic plates in the orange is small.*

The plate may be developed along two distinct lines. In the ordinary or "chemical" development the aim is to reduce completely the whole of the silver in a grain which has been affected by light, and to leave the remaining grains unaffected, for subsequent removal in the fixing bath. Unfortunately the present methods of development do not secure this result, the imperfections of development are discussed further below, for they are responsible for some of the most serious errors of photo-

* *Jour. Opt. Soc. Amer., 12, 1926, p. 401*

graphic photometry. The alternative process of development, termed for distinction "physical", aims at depositing silver from the developer on to the affected "centres" of a grain and at removing the unaffected parts, which comprise all but a very small fraction of its molecules. The grain of the resulting image depends therefore not on the size of the grains present in the original emulsion but on the duration of development, (which may be hours) and the composition of the developer. A perfected process of this kind would be ideal: at present, however, it appears to be less under control than chemical development, which is universally employed in astronomical photometry.

In "chemical" development the main difficulty lies in removing the products of development from the emulsion. These products (bromides) have the effect of slowing down the action of the developer, and since they accumulate most rapidly in the neighbourhood of dense images these are either under-developed (affected grains not entirely reduced) or else more weakly exposed parts of the plate are over-developed (unaffected grains attacked). This cause of error is known as the Eberhard effect,* and will be referred to frequently by this name in the course of this thesis. Over-development is especially undesirable in photographic photometry, since the images have to be measured in relation to the clear parts of the plate, and by over development these parts become unevenly fogged.

Several methods have been proposed for removing the harmful bromides or for avoiding their effects. The

* *G. Eberhard, Phys. Zeits. 13, 1912, p. 288.*

usual process of rocking the dish in which the plate is developing has the effect of distributing them to some extent, so that though uniformity of development is not attained, any small area not more than a few millimetres square having no very intense images may be taken as uniform. The effects of uneven development are then avoided in stellar work if an image is compared only with those in its immediate neighbourhood and symmetrically distributed around it. This method, which is used wherever possible by the writer in preference to others, may sometimes be secured directly; in other cases artificial comparison images may be scattered over the plate,* best by repeated exposures to a constant source, for equal images are difficult to realise. In other methods of development attempts are made to give the developer a rapid swirling motion. The use of a roller or brush in actual contact with the emulsion+ appears to involve too much risk in stellar work, and the most promising method seems to be that in which a bar is made to slide during the development across the surface of the plate, just clear of the emulsion.‡

Other sources of error due to the plates arise from variations in film thickness, from uneven drying after coating, from fog induced by conditions of storage, and from uneven drying after fixing and washing. If the images are dense they will be directly affected by variations in film thickness (see Appendix 1, Part VI), weak images are not directly affected, but will be influenced

*H. T. Stetson & E. F. Carpenter, Astrophys. J. 58, 1923, p. 36.
+O. Bloch, Phot. Jour. 61, 1921, p.425.

‡Dobson, Griffith & Harrison, "Photographic Photometry," Oxford, Clarendon Press, 1926, p. 77.

by the effects of the variations on the rates of development and drying. Uneven drying after coating may give rise to differences between plates cut from the centres and edges of the large sheets coated in the factory. Storage conditions might be considerably improved by avoiding the use of paper separators between the plates - these invariably affect the emulsion, probably by evolving vapours which resemble light in giving rise to a latent image. Some firms leave the emulsion intact when the plates are cut, and fold them into book form, the slight separation due to the bent gelatine being sufficient to prevent contact marks, while the circulation of vapours is in some degree hindered. The danger of contamination from such vapours is an argument against "conditioning" plates by keeping them under special conditions of humidity, since to be effective such measures must allow of the circulation of air. There is at present no way of drying plates uniformly - the most approved method appears to be to dry them face up in still air and to leave an ample margin, in fact, all the errors mentioned in this paragraph are diminished by using only the central region of a plate.

The final limit to the accuracy in the measurement of stellar images is set by what is called the "graininess" of the plate, the actual grains being too small to give rise to irregularities in normal cases. The subject is well treated (by A. C. Hardy) in a recent monograph* which is frequently referred to in later chapters. As he states (p. 27) the graininess of the

* *F.E. Ross, "The Physics of the Photographic Developed Image" Eastman Kodak Co. Rochester, N.Y., 1924.*

so-called clear portions of a stellar negative which are exposed only to scattered light and sky light show graininess to a much more disturbing extent than the images themselves, they show also an increase in graininess with concentration of developer and time of development. Some negatives submitted for recording*by the Koch photometer of the Royal Observatory, Edinburgh, (described in appendix 4) showed this latter increase strongly. In the normal way the graininess of the clear regions of a process plate requires a slit of the order 0.5 mm. × 0.005 mm. to render its effect apparent. The plates in question had been developed for about ten minutes in Rodinal 1:10 solution in order to obtain as much contrast as possible, and though recording was done with a slit about 8mm. × 0.03mm. the graininess is very marked, even with a process plate.

The results given in fig. 1 refer to conditions more closely resembling those found in stellar photometry. They correspond to the results of using a photometer spot (p. 11) with an area of about 0.04 sq. mm., rather less, that is, than has as yet been employed in the measurement of stellar images. The results are given on a uniform scale of percentage of the corresponding deflection - if allowance is made for the fact that the images *b* transmit less than one-third of the light of the clear parts *a* it will appear that the graininess shown by a continuous record does not vary greatly, measured in millimeters, over most of the useful scale. If, however, the oscilla-

* *Two of the records are reproduced in a paper by Dr. H. R. Robinson on "X-ray Terms and Intensities". Phil. Mag. 50, 1925, plate V.*

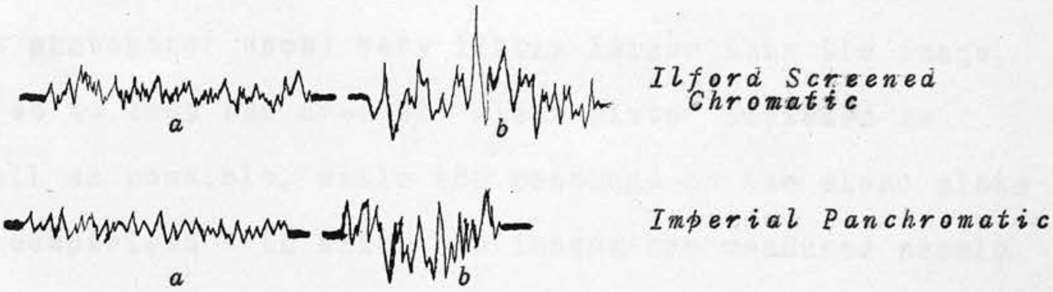


Fig. 1. Photometer records showing effect of plate grain. a, Clear plate; b, Image of density 0.5.

tions are converted into magnitude measures the effect is to multiply enormously those over the clear parts *a* relative to those over the image *b*. In these records, moreover, the emulsion is really "unexposed", sky fog and, it may be noted, invisible stellar images, are absent, otherwise the "graininess" would have been much enhanced.

Just as in the case of resolving power discussed by F. E. Ross in the monograph referred to on p. 8 , so in the case of graininess, the effect is so connected with the plate speed as to give, for a given flux of light, about the same accuracy of measurement whatever the speed of the plate, though this assumes that the image size may be reduced indefinitely and that the resolving power of the instrument is unlimited. Actually, however, it is rare to get the whole of a stellar image within an area less than 100μ in diameter;* while few automatic photometers can deal effectively with areas so small.

It appears, then that the best conditions for avoiding the effects of graininess are to spread the image over as large an area as is consistent with reasonable density, and to make the area included within the beam of light by

** By the whole of a stellar image is here meant that part of it which affetts an automatic photometer, the part visible to the eye may have a diameter of no more than 20μ.*

which the measurement is performed (called hereafter the photometer spot) very little larger than the image, so as to keep the area of "clear plate" included as small as possible, while the readings on the clear plate by comparison with which the images are measured should be multiplied until the error introduced through them is made small.

II.

THE MEASUREMENT OF THE IMAGE

In principle the measurement of a stellar image on a photographic plate is extremely simple. A beam of light is sent through the plate, the area illuminated being named conveniently the photometer spot, and the light absorbed from it by the image is measured. In the manner in which the light is measured the various photometers differ considerably; falling however into only two classes, namely, equalising or null photometers and deflection or automatic photometers. The photometers of the former class have been in use for many years. In them the beam of light which has passed through the deposit is made equal, usually by moving an absorbing wedge placed in one or other beam, to another beam of light which reaches the comparison device directly from the lamp. The beams may be compared by eye, thermopile, selenium cell or photo-electric cell, successful instruments of all these types being at present on the market, though not as a rule in a form suitable for stellar image work.* In the older instruments of this type, the Hartmann being a typical example, the eye was of course

** For a description of the original Hartmann (eye) instrument see Zeits. für Inst. 19, 1899, p. 97, Messrs Kipp & Sons (London agents, A. Hilger) make a null thermo-electric photometer. A selenium cell photometer has been designed by the British Photographic Research Assn. and is sold by W. Watson, London. The Cambridge Sci. Inst. Co. have recently placed on the market a photo-electric photometer designed by Dr. G. M. B. Dobson.*

employed, and the replacement of the eye by an electrical device has merely improved the accuracy and eliminated eye-strain. In so far as they have made possible accurate work at lower densities electrically worked null photometers raise the same questions as those of the deflection type, though in a less degree. Of deflection photometers there are but two successful types, namely, the Koch photo-electric photometer* and the Moll thermo-electric instrument.+ The latter is essentially the same instrument as was used by H. T. Stetson‡ for stellar photometry - its use in sensitometry dates back to the early days of the dry plate - but the improvements in the thermopile and galvanometer are largely due to Dr. Moll.** In this instrument the beam of light which has passed through the photographic plate is focussed on to a thermopile connected to a short period galvanometer with lamp and scale, the readings of which give the measure of the light passed. Unlike instruments of the equalising type, this form of photometer cannot be made free from the effects of variations in the source of light, and its accuracy depends almost entirely on the constancy of the source. The thalofide cell, which is sensitive

*P.P.Koch, Ann. der Physik, 39, 1912, p. 705. Goos, Zeits. für Inst., 1921, p. 313

+See lists of Messrs Kipp & Sons, Delft.

‡H.T.Stetson, Astrophys. Jour. 43, 1916, pp. 253, 325; and Popular Astronomy, 31, 1923, p. 253.

** Phys. Soc. Lond. Proc., 35, 1923, p. 253.

to the infra-red has been used in place of a thermopile, a photo-electric cell might also be used; but since variations in the effective temperature of the light source affect the short wavelengths to a much greater extent than the long, it follows that in respect to freedom from effects of this kind the thalofide cell, which has its main region of sensitiveness at wavelengths of the order of 10μ, is preferable to the thermopile, which again is preferable to the photo-electric cell.

In order to avoid the effects of fluctuations in the light source the Koch photometer is constructed on a different principle. Two photo-electric cells are connected to a high potential battery and to a thread electrometer as shown in figure 2. (Koch's original diagram shows a third cell; but this, as he states in his description, is not an essential part of the arrange-

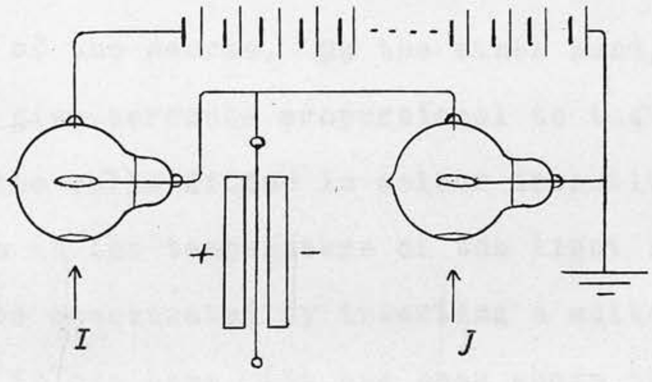


Fig. 2. Scheme of Koch photo-electric photometer.

ment.) It is readily shown that if the current through a photo-electric cell is proportional to the illumination the deflection of the electrometer in the above scheme will depend on the ratio of the illuminations of the two cells, and not on their absolute values. For if V is the potential of the cell battery and v that of the thread of the electrometer, the current through the one cell will be $I.f(V - v)$, while that through the other will be $J.F(v)$, I and J being the illuminations on the cells. Since these currents are equal when the thread has taken up its equilibrium position, it follows that v depends on the ratio of I to J, and not on their absolute values.

The beam passing through the photographic plate is thrown on to the cell which is connected directly to the high potential, the other being illuminated directly by the same source of light. Even for cells which always give currents accurately proportional to the illumination it does not follow that temperature fluctuations of the source will be entirely eliminated, for unless the deposit on the plate is perfectly neutral in its colour the two illuminations will be modified in different ratios owing to the change in the colour of the source. On the other hand, if the cells do not give currents proportional to the illumination or if the cells differ in colour sensitiveness, small changes in the temperature of the light source may still be compensated by inserting a suitable colour filter in one beam. (It has been shown that a photo-electric cell may be expected to have a different colour sensitiveness at a low voltage compared with that for high voltages across its terminals.*) That some photo-electric cells do give currents which are very accurately proportional to the illumination is shown in chapter VI; vacuum cells properly designed are even better in this respect.†

*H.E.Ives and T.C.Fry, Phys. Rev. 20, 1922, p. 112.

†See the description of the cells made by the General Electric Co. dated May, 1924.

A large instrument on the same lines as the one described by Koch has been set up to designs by the writer* at the Royal Observatory, Edinburgh, (see appendix 4) from components, some of obsolete instruments, others, such as gearing, made to order by outside firms, and others again made in the Observatory workshop, the writer himself making the photo-electric cells. The instrument used at the same Observatory for measuring stellar images was also constructed in the Observatory workshop to the writer's design,* (the plateholder was made by Messrs A.H. Baird, of Lothian Street, Edinburgh); its construction and use in sensitometry are described in appendix 1, part I, but some further details may be given as to its use in stellar photometry. A discussion of the three distinct ways in which this instrument may be used for the purpose will assist in making clearer the relative advantages and disadvantages of the different kinds of photometer.

In the first place, the instrument may be used as an equalising or null photometer (using the term "null" in a rather broad sense) by inserting a moveable wedge of neutral tinted glass at either P or N (appendix 1, part I, fig. 4) and moving this wedge until the thread is brought to a fixed position. The position of the wedge would then give a measure of the absorption of light taking place at the pinhole H. In the second place, the arrangement described in appendix 3 p. (347) may be used, as in the photometer described in appendix 4,

* *It should be mentioned that in the preparation and execution of these designs they were considerably improved as the result of suggestions made by Professor Sampson, by Mr. J. McPherson, the Observatory instrument maker, and by Mr. J. Ritchie, of Leith Street, Edinburgh.*

to obtain a uniformly divided transparency scale, which is given by thermo-electric deflection photometers. Finally, Koch's original arrangement may be adhered to, the result being a deflection instrument with its deflection nearly proportional to the square of the transparency.

The reason for rejecting the null form is not on account of its scale, for there is no reason why the wedge, which gives a uniform scale of density or log(transparency) should not be replaced by an arrangement giving any other kind of scale; but on account of the loss of time in making readings of the clear plate near each image. The method of reading by moving the electrometer plates described in appendix 3 was rejected for the same reason, in spite of the long scale which could have been obtained by the addition of a spiral drum to the plate screw. The advantage of having a scale divided according to the square of the transparency rather than to the transparency itself is seen when the translation of the results into magnitudes is considered.

Taking first the case of focussed images, discussed in chapter III, it is shown there that the diameter of the image bears a nearly linear relation to the magnitude. It is convenient, therefore, to work in terms of the "effective diameter", that is to say, the diameter of an opaque round spot of equal absorbing power to that of the image concerned. It is then found that in some cases the effective diameters of ill-defined images bear a linear relation to the magnitude - an example of this is given in chapter V, (p. 49). Figure 3 shows the scales of effective diameters corresponding to (1) a uniform

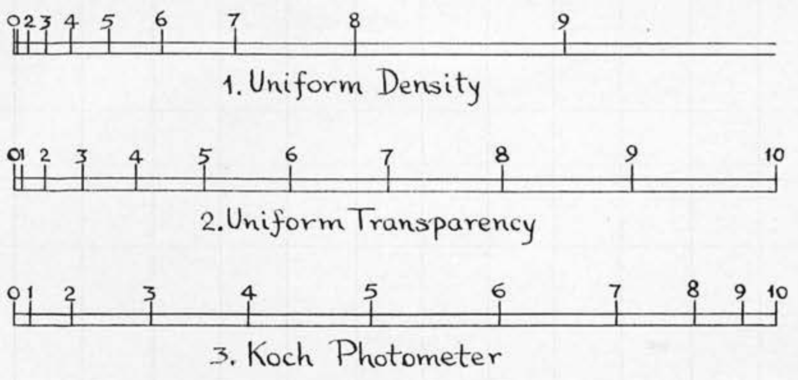


Fig. 3. Scales of image diameters, Light spot = 10.

density scale, (2) a uniform transparency scale and (3) the scale of the stellar image photometer described in appendix 1, part I. In these scales the diameters are given in tenths of the diameter of the photometer spot, in the typical instance of focussed images given in chapter III. p. 25, the increase in diameter per magnitude with a yellow filter is found to be 30μ, so that the numbers on the above scales will represent magnitudes in that case provided that the photometer spot is 300μ in diameter, a very usual case. It will be seen that the scale of the Koch photometer leads to a much more open scale in the case of the smaller stars, the gain being equivalent to nearly two magnitudes.

With sharply focussed images it is apparent that the accuracy of measurement will be great when the image takes up a considerable part of the area of the photometer spot. To cover a large range of images the size of the spot may be varied, and here the very great sensitiveness of the Koch photometer is useful, for it allows of the use of photometer spots as small as 100μ if desired. The smallest spot used in the writer's instrument has a diameter of 180μ; that used in the photometric investigation described in chapter VIII was 240μ in diameter, which is perhaps the most useful size where the images are not well defined. A spot of over 300μ in diameter would be suitable only for comparatively large images.

The method used by the writer for isolating these small areas is very similar to that used in the pioneer work by Stetson* at the Yerkes Observatory, that is to say, the employment of a diaphragm of the area required placed almost in contact with the emulsion. The large relative apertures of the condensing and receiving lens system used by Stetson necessitated the actual contact of the diaphragm and emulsion if errors from scattered light were to be avoided. In the apparatus used by Schilt,+ which is also thermo-electric, it has been thought sufficient to project on to the plate the image of a small diaphragm, and to project this again on to a diaphragm placed at the thermopile; but it appears in a recent report‡ that the slight imperfections of focussing caused by the lack of flatness of the plate may be sufficient to introduce appreciable errors. The writer, with a photo-electric instrument, is more favourably situated, for the great sensitiveness of the instrument allows the aperture ratios to be kept so low that errors caused by the variation of the distance between emulsion and diaphragm are completely negligible, and actual contact is unnecessary. On the other hand, the actual isolation obtained by the method used by Schilt has proved in the writer's hands inferior to the diaphragm

* *H. T. Stetson, Astrophys. Jour. 43, 1916, p. 253, & 325; and Popular Astronomy, 31, 1923, p. 253.*

\+ *Bull. Ast. Inst. Neth. II, 1924, No. 60. Groningen Publications, 32, 1924.*

‡ *Bull. Ast. Inst. Neth., III, 1926, No. 92.*

method, possibly because the lenses employed were not specially corrected in the region of the spectrum to which the photo-electric cell is most sensitive. This point will be returned to later.

As is pointed out in chapter IV, uniformly illuminated images may be measured by two distinct methods, the spot being in the one case sufficiently large to more than cover the image and in the other so small as to be completely covered by it. In the latter case the results given in appendix 1, parts II and IV, show that the magnitude bears a nearly linear relation to the quantity $log_{10}\frac{1-T}{T}$, where T is the transparency of the image. Scales of this quantity corresponding to those of the effective diameter of fig.3 are shown in fig. 4 The factor by which the numbers given have to be multiplied to give a scale of magnitudes is normally about 1.25, and a comparison of the scales will show the characteristics of this method of measurement, namely, great accuracy coupled with small range. In chapter IV reasons are given for preferring the use of images smaller than the light spot, in this case spreading of the image at its edges will be an important factor with the denser images, and the corresponding scales will be intermediate between those of figs. 3 and 4.

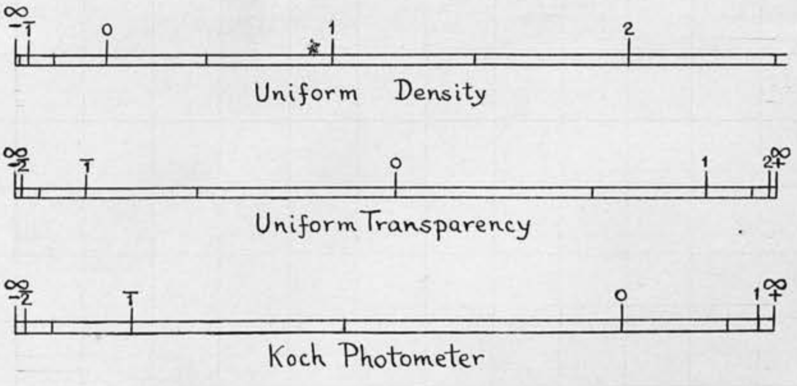


Fig. 4. Scales of $\text{Log}_{10}\frac{1-T}{T}$

Much remains to be done in designing a photometer suitable for measuring stellar images. At present the high sensitiveness of the Koch instrument and its freedom from effects of lamp variations appear to make it the best type for the purpose. Its speed could be improved by the use of vacuum cells filled with Sodium to give high sensitiveness coupled with freedom from the effect of the red end of the spectrum. A good view of the field could then be secured by making the diaphragm limiting the photometer spot of red or orange glass. The Lindemann electrometer appears to be the best on the market for the instrument - the writer's form, however, is easily made and quite effective. (See appendix 3). The only drawback is the necessity for a high potential battery. It would be worth making experiments to see whether by obtaining the voltage supply from the same source as is employed for the lamp current an arrangement could be obtained which would be independent of changes in the voltage. The Pointolite lamp, for instance, gives a lower arc voltage for a higher terminal voltage, so that at some point of the resistance the voltage must be approximately constant. With direct current it would not be possible to use this feature of the lamp to provide the voltage for the cell and electrometer without either making the case live, or having a double casing. There seems no reason, however, why the arrangement shown in Appendix 1, part I should not act with alternating current, in which case a small transformer could be used over the steady voltage points, or alternately, a gas-filled lamp of the filament type could be used, and the arrangement compensated by a light filter in front of one cell.

III.

SHARPLY FOCUSSED IMAGES

The errors of stellar photographic photometry depend very greatly on the manner in which the beam of light from the star is distributed when it reaches the plate. In most of the work on this subject the distribution has not been specified with any approach to accuracy, and where in addition the measurement of the resulting images is made by eye it becomes difficult, if not impossible, to form any definite ideas as to the effects of suspected sources of error. Where, on the other hand, the light falls on to the plate in a definite manner and the image is measured by photometer, it is not difficult to trace the effects of different sources of error, and to find conditions under which they vanish.

There are two ways of distributing the light which may be regarded as ideal; the one being to spread it as uniformly as possible over a definite area of the plate, the other to focus it on to an area so small that, in comparison with the size of the image formed by scattered light from it, the area may be taken as a point. These ideal conditions, as might be expected, are in neither case easily realised: there are, however, some practical cases in which they are approached very closely. In this chapter the results of applying the light to a very small area, of the order 10μ in diameter, will be discussed, the next treating of the uniform image.

It is impossible to treat the subject of focussed images without continual reference to the able manner in which it has been handled by F. E. Ross, even though his treatment* has reference to eye measurement. It will appear shortly, in fact, that when the light from the star does fall on an area small enough to be regarded as a point, the image formed is dense up to the edges, so that it may be treated almost as an opaque circular patch, for which the results of eye and photometer measures would be identical.

Ross finds it impossible to give an exact mathematical treatment of the growth of the image with exposure, but he shows that the assumption that the scattered light falls off in accordance with Bouguer's law

$$I = I_0 e^{-kx}$$

where I is the light intensity at a distance x measured perpendicularly from the edge of an illuminated area, I_0 the intensity just within the edge, and k a constant, leads to the law of image growth used by Scheiner+

$$m = a - b.d$$

where m is the magnitude of a star, d the diameter of its image, and a and b plate constants. He finds this formula to satisfy his observations of small images, and considers the use of Scheiner's formula justified up to image diameters of about 150μ. Beyond this he finds some compromise necessary between the law of Scheiner and that used at Greenwich, viz.-

$$m = a - b\sqrt{d}$$

and proposes the use of a formula involving an additional constant,

$$m = a - b\sqrt{d + h}$$

On the assumption that a law similar to Scheiner's holds when the time of exposure is varied instead of the

* *F. E. Ross, "The Physics of the Photographic Developed Image," Eastman Kodak Co. 1924.*

\+ *"Die Photographie der Gestirne", Leipzig, 1897.*

illumination, Ross calls the increase of image diameter obtained by doubling the duration of exposure the "turbidity" of the emulsion. (Allowing roughly for the deviations from the reciprocity law, the constant *b* of Scheiner's formula may be taken as about 1.5 times the turbidity.) The turbidities he finds by laboratory experiments range from 5μ to 20μ, being, as might be expected from the transparency of an undeveloped emulsion, smallest in light of short wavelength, and greatest in the yellow.

When he compares the results found for stellar images at the Yerkes Observatory with these figures, Ross finds good agreement, and concludes that the whole of the image growth is due to the spreading of light through the emulsion, the effect of aberrations being negligible. It is somewhat unexpected to find that the aberrations of a refractor, the peculiar diffraction effects present even at the centre of the field of a reflector, and the unsteadiness of the atmosphere, produce no appreciable effect on the rate of image growth. If, however, the result is considered in the light of the statement made on p.2 regarding the way in which the eye is attracted by a sudden fall of density, while a gradual fall passes unnoticed, it appears likely that this state of affairs will cease to hold good for automatic measures of the images.

An instance of this is found by comparing the turbidity given by Ross*for the Yerkes 6 inch U.V. camera, viz. 20μ for Cramer Iso. plates and yellow light, with that derived from the thermo-electric measures of images

* *Loc. cit., p. 101*

given by the same camera, plates and light, published by H. T. Stetson.* The increase per magnitude in "equivalent diameter" obtained from Stetson's figures is about 50μ, corresponding to a turbidity of 33μ. The difference cannot be attributed to the rather large diameters of Stetson's images, (from 120μ to 280μ) since Ross finds that for this camera the law of Scheiner applies over a wide range. Neither is it likely that the lack of sharpness of the edges of the images given by the perfectly focussed image is concerned. For a turbidity of 20μ, the value of k in the expression

$$I = I_0 e^{-kx}$$

given above is (assuming the reciprocity law to hold) $\frac{\log_e 2}{20}$, leading to a fall of illumination in the ratio of about 29 : 28 per 1μ. The gamma of an emulsion of this kind with specular illumination is about 3, hence the resulting fall of density per 1μ at the edge of the image is about 0.05, and an image 200μ in diameter will fall off by one unit of density in one-tenth of its diameter.

That cases do exist where even with automatic measurement aberrations, &c. are negligible is seen from the following instance. There is affixed to the 15 inch refractor of the Royal Observatory, Edinburgh, a Zeiss "Tessar" lens of 59 cm. focal length and an aperture of f:6.3. This lens was not made for astronomical use and consequently its corrections in the ultra-violet are inferior, but in the yellow and green it gives extremely small images, while owing to its short focal length atmospheric unsteadiness even in the Edinburgh climate does not produce noticeable effects. With panchromatic

* *Astrophys. Jour.* 43, 1916, p. 272.

plates and a Wratten K2 filter its turbidity for small images is 20μ, rising to 25μ or more for images above 150μ in diameter, and this shows no difference whether the images are measured by eye or by photometer. When a blue filter is employed with the same plates the effect of aberrations shows up strongly, and for automatic measures of equivalent diameters the turbidity is upwards of 50μ. The figure shows a positive print from a negative taken by the writer with this camera, magnified

(*a*)
(*b*)

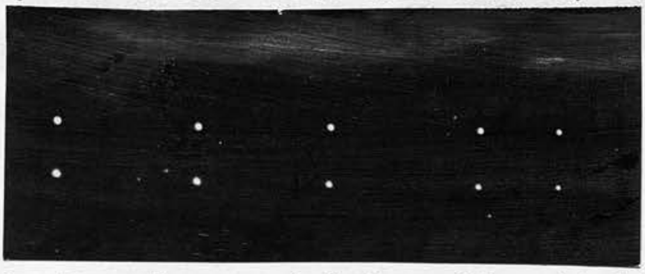


Fig. 5. Images (a) in yellow light; (b) in blue light, given by Zeiss Lens, f:6.3. × 7

seven diameters, showing two series of images of the same star with various exposure times, the upper row (*a*) being taken through a yellow filter and the lower (*b*) through a blue filter.* As an illustration of the difference between eye and automatic measures, it will be noticed that to the eye the smallest image of series (*b*) appears smaller than the corresponding image of (*a*). To the photometer, however, the opposite is the case: thus, the measures of the two smallest images are

	In yellow light	*In blue light*
Measured by eye	*150*	*120*
" " photometer	*149*	*168*

It might be expected that the light passing through the image would cause the effective diameter measured photometrically to be smaller than the diameter measured

* *The images are not in the centre of the field, and accordingly show slight coma.*

by eye. Since, however, the density of the image over the greater part of its area exceeds 2.0, this part therefore transmitting less than one-hundredth of the incident light, the effect on the photometer of the light passing through the image is small. A more potent cause of reduction of the effective diameter is the Eberhard effect, (p.6), the development products of the dense image so weakening the effect of the developer in its immediate neighbourhood as to make these clearer than the unexposed plate. For this reason focussed images give an even sharper edge than that calculated above, and the effect of weak aberrations is nullified.

We next proceed to examine the effects of small variations from the ideal conditions. First it may be remarked that uneven development has very little influence on the measures, provided that development is not so weak as to leave the images grey rather than black. To the photometer it matters little whether the dense parts have a density of 1.5 or 2.5 - in either case so little light passes that they may be regarded as opaque.

The effects of defective focussing are more serious. Images of the nature discussed here cannot be formed in practice by long focus objectives, owing to atmospheric effects; and their only application is to images given by short focus lenses, and even here over only a comparatively small field compared with that for which such lenses are usually computed. To be of use in stellar photometry these lenses must have large aperture ratios, and in consequence have a very small depth of focus, so that even over the best part of their field a slight tilt of the plate, or even deviations from flatness on the part of the emulsion, are sufficient to give rise to appreciable errors. To show the effects of imperfect focussing, even at the centre of the field where the image remains symmetrical, the results of a test plate taken by the writer with the Zeiss lens mentioned above, using a yellow filter and a panchromatic plate, are given below. The deduced magnitude error is given on the assumption of a turbidity of 20μ found under these conditions, corresponding to a difference of diameter of 30μ for unit difference of magnitude.

Distance of plate from focus. mm.	*Equivalent diameter of image mm.*	*Deduced error, magnitudes.*
0.9	0.137	1.07
0.6	0.128	0.77
0.3	0.113	0.27
0.0	0.105	--
- 0.3	0.117	0.40
- 0.6	0.134	0.97
- 0.9	0.149	1.47

Even with the moderate aperture of f:6.3, then, a tilt of the plate of one-fifth of a millimeter over the diameter of the field used will introduce errors of the order of one tenth of a magnitude. Lenses of aperture f:2 are now made and may find a use in astronomy; but the difficulties in the way if they are applied to stellar photometry are very great.

Even more serious disturbances occur if the guiding is defective and the images are in consequence slightly elongated. Effects of this kind rendered it impossible to use the 24-inch reflector of the Royal Observatory, Edinburgh, for stellar photometry, at any rate when in focus, the driving arrangements being imperfect. The effect of guiding error is likely to be most injurious when a field of stars is compared with the North Polar Sequence, for in the latter case the error will probably vanish. In the next chapter a comparison is made between the effects of what corresponds to an excessive case of bad guiding on sharply focussed and on evenly illuminated images, the former being much the more serious. It is obvious from the figures given above that the use of the automatic photometer does not always eliminate the corrections which have normally to be found for different parts of the field of a lens. The conditions under which its use renders these corrections small are discussed in chapter V. The images measured by Schilt, whose results form the basis for some rather too optimistic statements on this point,* are not of the nature of sharply focussed images, as the term is employed here.

It has already been noticed that the turbidity of an emulsion depends on the wavelength of the light, the longer wavelengths being transmitted through the undeveloped emulsion more readily than the shorter. Ross+ gives the figures 6μ for wavelength $400\mu\mu$, 11μ for wavelength $480\mu\mu$, and 15μ for wavelength $640\mu\mu$, but it is doubtful whether lenses exist which are sufficiently free from chromatic aberration to give rise to sharp images in all these wavelengths. Assuming their existence, however, it is evident that with heterochromatic light the rate of growth will finally depend on the long wavelengths. According to Ross, the only region where the turbidity does not vary with wavelength is the

* *Int. Ast. Union Trans. 2, 1925, p.85.*

+ *Loc. cit. p.107.*

orange and yellow.

Some notes on the possible accuracy which can be reached with perfectly focussed images may be added. Ross, assuming a turbidity of 10μ, corresponding to a growth of 15μ per magnitude in the image diameter, came to the conclusion that since in measurement by eye a difference of 1μ is the utmost that can be perceived the accuracy is too low for good work. The photometer will measure to 0.1μ if desired, but the error due to graininess (p.8) has to be considered, both in the image and in the surrounding plate. With the focussed image the photometer spot should be small for the smaller images, otherwise the accuracy of measurement is low; but these conditions lead to large graininess error. As an example of the best that can be expected a series of seven images of the same star, each exposed for 30 seconds, may be quoted. The Zeiss Tessar lens was used at its best focus, with a yellow filter, the difference in diameter for one magnitude being, as before, 30μ.

Image	*Equivalent diameter* *mm.*
1	0.1568
2	0.1595
3	0.1528
4	0.1569
5	0.1560
6	0.1537
7	0.1569

Mean deviation from mean = 1.6μ

Deduced probable error of one observation = $0^m.045$

A comparison of these results with those taken under exactly similar circumstances with a blue filter will show that the error is not due to errors of timing or of atmospheric changes, and by far the greater part seems to be due to graininess. If this is so, a turbidity of 10μ instead of 20μ, would increase the error to at least 0.06 mag. for a single image.

It is concluded, then, that if perfectly focussed images were practicable the utmost accuracy that could be expected for images about 150μ in diameter is $0\overset{m}{.}05$ for a single image measured by photometer; that any small deviations from perfect focussing, whether due to plate tilt, aberrations or imperfect guiding, will introduce errors which cannot be eliminated in the measurement of the images; and that the only region of wavelength where heterochromatic photometry will give consistent results (see p. 63) is in the orange and yellow, a region so narrow that it can scarcely be termed heterochromatic. Their advantages are that they are not appreciably affected by irregularities of development, and that in a field of stars photographed at a single exposure the range of images measurable is unrestricted.

I V.

UNIFORMLY ILLUMINATED IMAGES

In dealing with sharply focussed images in the preceeding chapter the illustrations were all taken from stellar observations. The case of uniformly illuminated images is more easily investigated in the laboratory, where the images may be of considerable area, thus eliminating the error due to the graininess of the plate. Where factors such as the Eberhard effect or halation (the effect of scattered light in the emulsion or of reflections from the back of the plate) are present it is necessary to use large images to obtain consistent results.

The measurement of such uniformly blackened areas falls within the province of sensitometry, from which the nomenclature has been carried into astronomical work. The transparency T of a deposit is defined as the ratio of the light transmitted by it to that transmitted under similar conditions of illumination by an unexposed part of the negative. As might be expected, T depends to some extent on the illumination employed. The colour is of little importance; but owing to the light scattered by the deposit the transparency is much influenced by the relative apertures, giving rise to two idealised systems of measurement. In the "specular" system both beams are normal to the plate and of small relative apertures: in the "diffuse", on the other hand, while one beam is small and normal, the other includes the whole solid angle on its side of the emulsion. The specular system gives the greater contrast and is therefore preferred in astronomical work: the diffuse is usual in sensitometry from its relation to the conditions applying in contact printing. The density D, which like the transparency may be either "specular" or "diffuse", is defined by the relation

$$D = -\log_{10} T$$

As an example of the difference between the two systems, it may be mentioned that a diffuse density of 1.0 corresponds as a rule to a specular density of about 1.5.

The best known relation between the density of a uniformly illuminated image and the exposure which produced it is that used in the Hurter and Driffield system of sensitometry, and though it is based, not on the facts, but on the conditions necessary for the preservation of correct contrast in pictorial photography, it is sometimes used in photometry as a convenient empirical formula. The assumed relation is

$$D = \gamma \log_{10} E/i \qquad \text{.............} \qquad (1)$$

where E, the exposure, is the product of the illumination I and the time t of exposure to it. In this chapter t is supposed constant. The constants γ and i are termed the gamma and the inertia respectively. In practice the curve obtained by plotting the density against the logarithm of the exposure has a point of inflexion at a density depending on the emulsion, and over a considerable range of density around this point the above relation holds with some precision. The use of the terms gamma and inertia is confined to this region, and in the low density region or "under-exposure" portion of the density range, where formula (1) ceases to hold good, they bear no relation to the properties of the plate.

To the eye this region occupies a very insignificant part of the full range of density, (up to perhaps 0.3 in an emulsion with a total density range up to 2.5); but to the automatic photometer this region is most important, for, as was seen in chapter II, it may occupy more than one-half of the useful scale. Where observations of density include this low density region Abney's formula

$$D = K\ (\log E/E_0)^2 \quad \ldots\ldots\ldots\ldots \quad (2)$$

(see appendix 1, part VI) is sometimes useful. When working with a deflection photometer it is more convenient to use the transparency than the density, and the writer has found the formula

$$\log_{10}\frac{1-T}{T} = Q \log_{10}E/E_0 \quad \ldots\ldots\ldots \quad (3)$$

in which Q and E_0 are constants, more useful than either (1) or (2). Examples showing the accuracy to be expected from the use of the formula (3) will be found in appendix 1, parts II and IV, from which it appears that Q in the low density region is about 2.0.

When applied to stellar photometry these formulae become, (1) $m = a - b.D$; (2), $m = a - b\sqrt{D}$, and (3), $m = a - b \log_{10}\frac{1-T}{T}$, m being the magnitude of the star and a and b plate constants. These formulae are directly applicable only to the measurement of images which more than cover the photometer spot. As will be seen directly, these are not the most favourable conditions for measurement: it is preferable to use a light spot large enough to include the whole of the image. Under these circumstances the only formula likely to be of use is derived from (3), it is

$$m = a - b \log_{10} \frac{x}{c - x} \quad \ldots\ldots\ldots\ldots \quad (4)$$

where x is the fraction of the light *stopped* by the image, and c the fraction of the area of the photometer spot which it occupies.

In the case of sharply focussed images it was seen that variations in development produced no appreciable effects on the measures. On uniform images the effect of

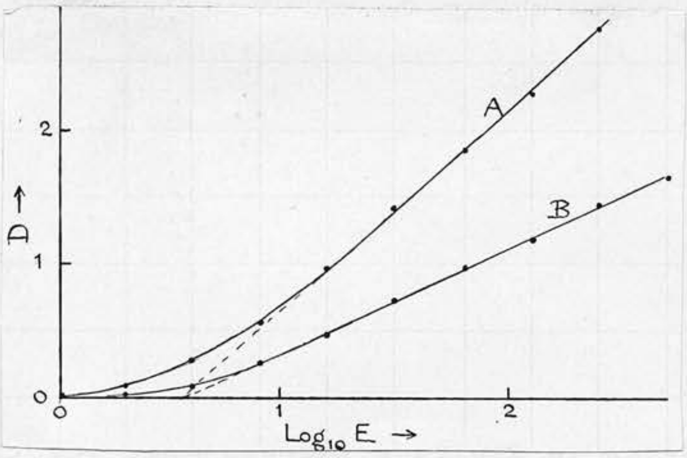


Fig. 6. Relation between density and log. exposure. Ilford Zenith plates, blue light.
A. *Developed 9 min.* B. *Developed $2\frac{1}{2}$ min.*

variations of development is very marked, and depends in in a somewhat peculiar manner on the density range used. Fig. 6 shows the observed (specular) densities of an Ilford "Zenith" plate (H. & D. 650) plotted against the logarithms of the corresponding illuminations (that is to say, against a uniform magnitude scale) and hence shows the density region to which formula (1) applies and the manner in which the constants of that formula are affected by development. It is evident that the constant i of the formula, corresponding to the constant a of the stellar formula deduced from it, is left unaffected, the change being confined to the constant y (or b). In fig. 7 the same observations are shown, but in place of the density the quantity $\log_{10} \frac{1 - T}{T}$ of formula (3) (called $\triangle$ in appendix 1) is taken. The difference between

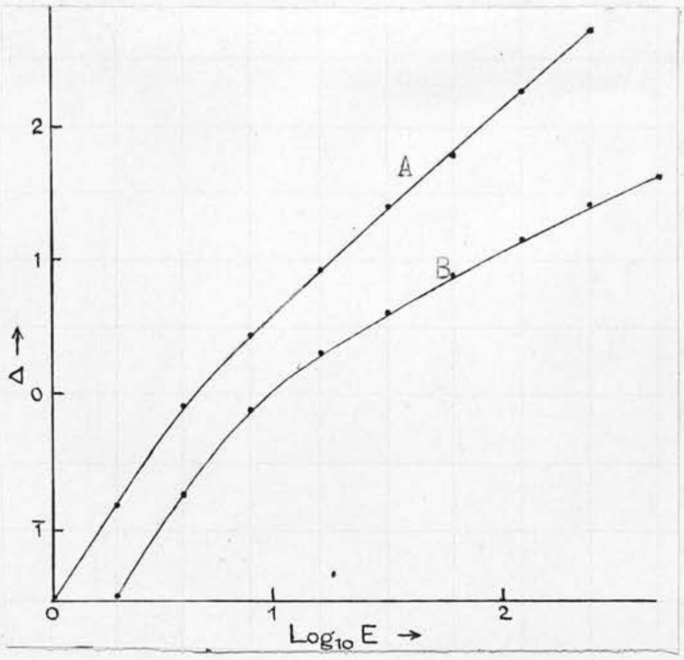


Fig. 7. As fig. 6, but with $= \log_{10}\frac{1-T}{T}$ *in place of* $D = \log_{10}\frac{1}{T}$.
A. *Developed 9 min.* B. *Developed* $2\frac{1}{2}$ *min.*

D and $\log_{10}\frac{1-T}{T}$ is small when the density is greater than unity, the high density portions of figures 6 and 7 are therefore practically identical, and for this region the use of formula (3) would involve Q as a term depending on the development, the quantity E_0 being independent of it. The difference in the low density region is striking, Q being independent of the development while E_0 (the index of the speed of the emulsion) depends on it. The figure, of course, shows an extreme case; with some emulsions at any rate it is possible to develop the low density parts so fully that any effect of development variations is confined to the higher densities, where even with the greatest precautions the error introduced may be serious. In the first paper by L. A. Jones and E. Huse on the relation between time and intensity in photographic exposure* development was carried out in a water-jacketed tray at constant temperature for a carefully measured time, and with carefully prepared

* *Jour. Opt. Soc. Amer. 7, 1923, p. 1079.*

solutions, yet the observations of a Seed 30 emulsion showed a systematic change of gamma with light intensity of 30%, which proved to be due, not to any actual effect of the kind, but purely to the differences between different lots of development chemicals. The methods adopted for overcoming the difficulty are given in later papers by the same authors;* the instance, however, serves to show what serious errors may occur in high density work, to which eye photometers of the Hartmann type are restricted by reason of their inaccuracy in the lower densities.

Small deviations from the ideal form of image may occur either through a simple increase of area, such as would be introduced by variations of focal length in extra-focal work; or they may cause varying illumination over the image. Where, as in the *Göttingen Aktinometrie*, the photometer spot is smaller than the image, any increase in image area necessarily entails a diminished effect on the photometer. Where, however, the photometer spot is larger than the image, there will be some particular size of image for a given flux of light from the star which will give rise to a maximum effect on the photometer beam, which effect will then be unaffected by any kind of small variations in the distribution of the light from the star. The density of this favourable image is found by considering the effect of an alteration of area, leaving the illumination uniform.

Let c be the fraction of the area of the photometer spot occupied by the image. If during the exposure the illumination over the image, supposed uniform, is I, and the total flux of

* *Jour. Opt. Soc. Amer. 11, 1925, p. 319; and 12, 1926, p. 32*

light over the image is constant, the product Ic will be constant, and

$$\frac{1}{I}.dI + \frac{1}{c}.dc = 0$$

If after development T is the transparency of the image, the light it absorbs from the photometer beam is say x, where x is proportional to $c(1 - T)$, hence

$$\frac{1}{x}.dx = \frac{1}{c}.dc - \frac{1}{1 - T}.dT$$
$$= 0$$

if x has its maximum value. Now introduce the quantity q defined in appendix 1, part I, p.(167), observing that the quantity Δ of that appendix is the same as the quantity $\log_{10}\frac{1 - T}{T}$ of formula (3) of this chapter, it follows that

$$\frac{q.dT}{T(1 - T)} = -\frac{1}{I}.dI$$
$$= \frac{1}{x}.dx + \frac{1}{1 - T}.dT$$

from the equations above, whence it follows at once that x is a maximum (or minimum) when $T = q$, 0, or, possibly, 1, which gives $x = 0$, Evidently the values 0 (perfectly focussed image) and 1, (infinitely spread-out image) give the minima, and q the intermediate maximum.

In appendix 1, parts II and IV, it is shown that q lies as a rule between 0.4 and 0.6. For values of T between these limits it is justifiable to neglect the effect of scattered light at the edge of the image, as has been done in the above analysis. The result that for images of a certain density the effects of small variations in the light distribution are eliminated by using a photometer spot larger than the image is of considerable importance, especially when by choosing this density of image it follows that the maximum possible effect is being obtained from a given light flux. The value of the density, from 0.3 to 0.4 in normal cases, is well suited to the scale of a deflection photometer, for when the image takes up a considerable proportion of the photometer spot the readings will be near the middle of the scale where the accuracy is greatest.

To give a set of figures showing the error which may be expected from variations of focus in extra-focal work, which should be comparable with those given in the case of focal images on p.28, the following set have been computed assuming the formula (3), with the value 2 for Q, for an image whose best diameter is 200μ, measured in a photometer spot of diameter 350μ. Under these conditions formula (4) becomes

$$m = a - 1.25 \log_{10} \frac{x}{.327 - x}$$

Neglecting the effect of scattered light, and assuming that the beam of light from the star may be considered as a cone of aperture f:6.3, the errors corresponding to changes of distance from focus are as follows:-

Distance of plate from true position mm.	Transparency of image	x	Deduced error m.	Cf. for focussed images m.	Cf. for small spot m.
0.9	0.898	0.100	+ 0.45	- 1.07	+ 1.18
0.6	0.826	0.124	0.27	0.77	0.85
0.3	0.702	0.150	0.09	0.27	+ 0.47
0.0	0.500	0.164	0.00	0.00	0.00
- 0.3	0.251	0.142	0.14	0.40	- 0.59
- 0.6	0.070	0.084	0.58	0.97	1.40
- 0.9	0.007	0.026	1.32	1.47	- 2.72

For a transparency of 0.007 it is certain that scattered light would lead to a considerable increase in the value of x for the last image, so that the error in practice would be much reduced in that case, though only when the light spot is larger than the image. Where the light spot is much smaller than the image the figures given in the last column show that the possibility of errors through slight variations of focus is very great.

Another comparison which may be made between the focussed and the uniform image is in respect to the error introduced by imperfect guiding. The case considered is not exactly a practical one; but the results must be exaggerrated instances of those occurring in practice, and it has the advantage of being easily worked out. The comparison is between the errors introduced by giving half the exposure on one area of the plate and half on an adjoining area far enough away to give separate images, but close enough for both to be within the photometer spot. In the case of focussed images, if the single image has an effective diameter d, the two will have each the effective diameter $(d - h)$, where h is the "turbidity" (p.24). The double image will therefore have the same effect on the photometer as a single image of diameter $(d - h)\sqrt{2}$, which differs in magnitude from a single image of diameter d by $\frac{(d - h)\sqrt{2} - d}{1.5\ h}$ magnitudes (approximately). Now h (p.24) is of the order 10 - 20μ, while d is seldom below 80μ, and ranges up to over 500μ. Assuming a turbidity of 15μ, the error will be half a magnitude for a single image 80μ in diameter; over two magnitudes for a moderately small image 150μ in diameter, and over eight magnitudes for a large image 500μ in diameter.

Compare these figures with the results for a uniformly illuminated image. Using formula (3) and putting Q equal to 2, the transparency T of the image is given (cf. appendix 1, part II, p.(183)) by

$$\frac{1}{T} = 1 + \frac{E^2}{E_0^2}$$

hence the light stopped by it is proportional to $\frac{E^2}{E^2 + E_0^2}$. It follows that two images, each having the exposure $\frac{1}{2}E$, are equivalent in absorbing power to a single image having the exposure E_1, where

$$\frac{E_1^2}{E_1^2 + E_0^2} = \frac{2E^2}{E^2 + 4E_0^2}$$

The difference of magnitude corresponding to the exposure ratio E_1/E is easily computed from this formula. For a weak image for which the transparency of the single image is 0.9 the difference is 0.34 magnitudes. It vanishes for an image rather denser than one of the correct density for maximum absorption for a given flux of light (p.38) and increases to an impossible amount as the image density rises. It may therefore be concluded that in practical cases the effect of bad guiding will be much smaller for uniformly illuminated images than for focussed images, and that in both cases it is likely to be serious for the dense or large images corresponding to the brighter stars.

The effect of varying the effective wavelength on the quantity y of formula (1), corresponding to the high density value of Q in formula (3), has been the subject of much controversy. Some remarks on the subject in appendix 1, part VI, may assist in clearing up the question, and if the conclusions are correct, it should be possible to obtain emulsions for which y does not vary with wavelength, it being merely a matter of employing a thinly coated plate. There is less doubt about the variation of Q with wavelength in the lower densities. The quantities Q of this chapter and q of appendix 1 may be regarded as reciprocals, and its dependence on wavelength deduced from the results given in parts II and IV of that appendix. As in the case of focussed images, the only regions in which Q is constant are those in the yellow and orange to which isochromatic and panchromatic plates are sensitive.

The chief difficulty which attends the use of evenly illuminated images is that of obtaining such images without undue increase of size. The unavoidable fifth order aberrations of short focus lenses (especially fifth order spherical aberration) lead to great divergencies from uniformity in their out-of-focus images, and it seems impossible to obtain approximately even illumination until the focal ratio falls below f:12. A photo-visual refractor working at about f:15 will produce suitable images over a considerable field, of not more than 0.3 mm. diameter. Extra-focal reflector images might be suitable, though the field would be small; but it will be seen in the next chapter that conditions for the use of large reflectors in focus are not unfavourable. The use of a *Schraffierkasette*, as in the *Göttingen Aktinometrie*, introduces other questions; some of these are considered in chapter VII, but they would require further consideration before applying the results of this chapter to such images. There is one other practical case in which a uniform image may be obtained: this is by the use of a very short focus lens placed at or just beyond the stellar focus of the telescope and having the photographic plate at its focus.* The result is to form an image on the plate of the telescope objective, and to throw on this image the light of any stars focussed within the aperture of the lens. Since the image formed is perfectly uniform and independent within wide limits of telescope aberrations, guiding errors, and wandering of the star

* *This idea is not original, it is, however, so obvious an application of the "Ramsden circle" of an eyepiece that it can scarcely be credited to any one person.*

the method should have great advantages in the study, for instance, of eclipsing variables which are too faint for photo-electric observations, but bright enough to be compared with other stars by a method involving separate exposures, without too great loss of accuracy from variations in atmospheric transparency. The method is also ideal for comparing the brightness of an extended object such as a major planet with an appreciable disc with that of a star. Here again the difficulty is to obtain small enough images for efficiency. An image of 200 μ diameter, which would be perhaps the best size, would require with a telescope of aperture f:12 a lens of focal length 2.4 mm.

To sum up, it appears that uniform images possess advantages over focal images in giving rise to smaller errors from small changes in the light distribution, such errors being eliminated for images of a particular density. The difficulty as a rule being to secure small enough images, graininess errors will be small, and since high aperture ratios are out of the question, errors from plate tilt or variations of focus will also be small. It is possible to obtain the maximum effect by their use from a given flux of light; but this possibility of high efficiency is usually off-set by the unduly large images which have to be used to secure uniformity of illumination. The accuracy of measurement is high when a deflection photometer is used; but the range of images measurable is restricted. They are subject to larger errors from variations of development than focussed images, more especially when the images are dense. Though actual figures cannot be given, an accuracy of $0\overset{m}{.}02$ should be within reach with images 0.3 mm in diameter.

V.

IMAGES OCCURRING IN PRACTICE

Of the two idealised forms of image discussed in the last two chapters, namely, the sharply focussed image and the uniformly illuminated patch, the first was found to be preferable in range of magnitude covered in a single exposure to a field of stars; the second in accuracy of measurement and economical use of the available light. Neither type can, however, be realised with any approach to perfection over a large field, while in the photometry of faint stars it is necessary to use large reflectors, so that the effects of aberrations, the wandering of the image due to atmospheric disturbances and imperfections of guiding, and the variations of focal length have to be faced.

In the case of both ideal image forms measurement by a photometer having a light spot larger than the image led to the elimination in certain cases of the effects of small changes in the distribution of the light from the star. In the case of the focussed image the effect is a minimum when the focussing is most perfect: the uniform image, on the other hand, gives a maximum effect when it is of a particular density. These results lead to the question whether it is possible to find a form of image which shall combine these two cases in such a way as to give measures independent of changes in the light distribution over a wider range than in either of the ideal cases discussed. It seems probable that the required form is an image containing a concentration of light accompanied by a fairly uniformly illuminated area - if so, the presence of spherical aberration and coma in the objective will be an advantage rather than a defect. There is another way in which an increase of effect with increased area of image may be partially compensated; this is by limiting the size of the photometer spot, so that it is little larger than the normal image, leading to "cut-off" with large images. This form of compensation may be carried still further by using in place of a uniform photometer spot, as has so far been taken for granted, a lighting distributed in such a way as to suppress the effects of growth without altogether eliminating them.

In photometry with lenses covering a considerable field of stars the most troublesome errors arise from variations in the distribution of the star's light over its image according to the position of the star in the field. Starting with the case of a telescope which gives well-defined though not indefinitely small images at the centre of its field, with aberrations increasing from the centre outwards, it may be confidently expected from the results of the last two chapters that the effect of the aberrations will depend on the magnitude of the star. Bright stars should give an increased effect as their distances from the centre increase, with a maximum when the mean transparency of the image is in the neighbourhood of the value of q for the emulsion, provided that the image is still small enough to be within the photometer spot. Further increase of image area with increasing distance from the centre will lead to a continuous fall in the measured effect, enhanced eventually by the effect of "cut-off". If the star is very bright, cut-off will come into action before the maximum is reached, and for a certain range of image sizes may balance the rise of effect with increased area. For much fainter stars it may happen that even the smallest images in the centre of the field are not dense enough to fulfil the condition for a maximum effect, the result will be that such stars will have their maximum effect at the centre of the field, falling off continuously towards the margin.

The large reflector offers an excellent example of these conditions, the aberrations increasing rapidly from the centre of the field outwards, and in the analysis by J. Schilt* of measures of plates taken with the 60 inch reflector of the Mount Wilson Observatory these expected results are seen to occur. Schilt gives his results in what he calls "galvanometer values", these being identical with the mean transparency of the area covered by the photometer spot, in parts per thousand. With images having galvanometer values of more than 700, that is to say, the faintest stars, there is a decrease in the effect for a given light flux in passing outwards from the centre. By the time a galvanometer value of from 600 to 500 is reached this decrease is replaced by an increase. Brighter stars, with images between 500 and 100 show no appreciable difference between centre and margins. Here, evidently, is the effect of compensation by cut-off, for an image absorbing 90% of the light will certainly lie very largely outside the photometer spot.

Schilt himself considers it dangerous to draw conclusions from his figures as to the systematic errors involved in the photometric measurement of stellar images;

* *Groningen Publication 32.*

but other writers have been less cautious in forming opinions and have even taken his results as showing that the method of measurement annuls the effects of aberrations. Though certainly much smaller than with eye measurement, the errors observed agree too well with those expected to be accidental, and cannot even in this favourable case be regarded as negligibly small.

It should be mentioned also that the more perfect the images at the centre of the field, the more liable they will be to errors from imperfect guiding. The 24-inch reflector of the Royal Observatory, Edinburgh, gives good images; but its guiding arrangement are inferior, and measures of some of the Harvard fields on plates taken with it gave the most divergent results for the denser well-focussed images when the photometer was used for the measurement. On the other hand, a plate of the Pleiades on which the fainter members of the Harvard Sequence were placed nearly a degree from the centre of the field gave for these members very consistent results, in spite of the fact that their shapes were such as to defy eye measurement.

Another instance of the effect of aberrations is afforded by the plates taken with the Zeiss lens mentioned in chapter III; but used without a filter on blue-sensitive plates. Under these conditions spherical aberration is noticeable, the images taking the form either of rings or of a central condensation with a hazy border. The effects of varying the distance of the plate from the focus are shown in fig.8a for a bright star and in fig.8b for a faint star taken at the same exposures. In the former case the images are all

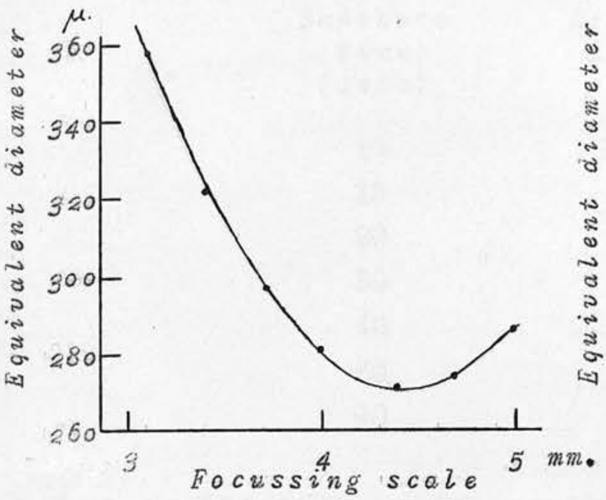


Fig. 8a. Effect of varying focus on large image

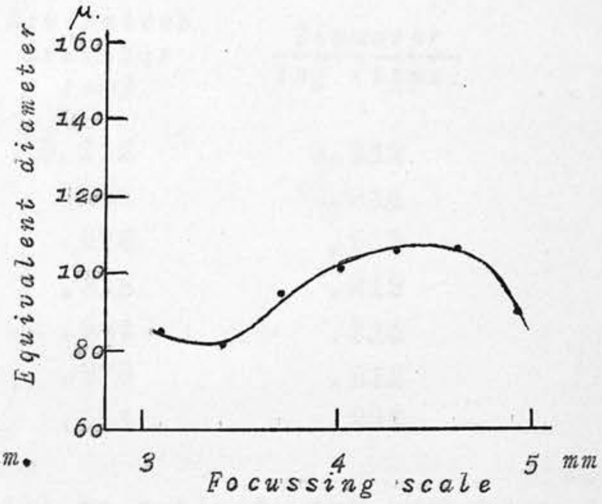


Fig. 8b. Effect of varying focus on small image.

too dense to give the maximum effect, hence all that appears is a minimum at the position of "best focus". The faint image is still bright enough to have a minimum between two maxima, only one of which is shown in the figure - in this case a small photometer spot was used in order to obtain greater accuracy, and the sharp fall from this maximum is to some extent the effect of cut-off. It will be noticed that the two minima occur at different positions of the plate - this is due, not to plate tilt, for the images were close to each other on the negative, but to the effect of the aberrations, which produce a different "best focus" for each size of image.

Comparing the results with those for sharp images in chapter III. the greatly increased rise of equivalent diameter with increase of exposure will be noted. The increase here for unit difference of magnitude is from 90 to 95μ, compared with only 30μ in the case of sharp images even in the favourable case of yellow filtered light. With the substitution of effective diameter for diameter the law of growth is, within the experimental error, that given by Scheiner, as the following measures show:-

Exposure time (secs)	*Equivalent diameter (mm)*	*Diameter / log (time)*
10	0.212	0.212
15	.251	.214
20	.276	.212
30	.315	.213
40	.344	.214
60	.379	.213
90	.404	.207

The high accuracy will be noticed; the probable error of a single image is rather less than $0\overset{m}{.}02$. Another example of the high accuracy to be expected in the presence of considerable spherical aberration is given below in a form exactly comparable with the measures of sharply focussed images on p. 30, i. e. for a series of seven equal exposures on the same star, close together on a single negative:-

Image No.	*Equivalent diameter (mm)*	
1	0.2347	
2	0.2338	
3	0.2369	
4	0.2332	
5	0.2338	*Deduced probable*
6	0.2338	*error of one observation* = $0\overset{m}{.}013$
7	0.2361	

The images are perhaps larger than the bulk of those met with in photometric work; but even so the accuracy attainable in photographic photometry does not seem to have been realised. Most astronomers have come to regard the photo-electric cell as greatly superior to the photographic plate in accuracy - this in the writer's opinion is untrue. It is admittedly superior in the manner in which it obeys the reciprocity law, so allowing more convenient methods of working; but the advantage which it enjoys at present in the accuracy of the results obtained is due almost entirely to the conditions under which it is employed - conditions more favourable to accuracy than have ever been applied to the photographic plate. Take, for instance, a twelve-inch telescope, with a small diaphragm (which in this case might be replaced by a short focus lens as on p.42) at the stellar focus, into which the image of the star is brought; place immediately beyond this a photographic plate surrounded by an enclosure kept at constant humidity; and confine the observations to stars brighter than the 6th magnitude, giving exposures of the order of 20 seconds. The results would then afford a fair comparison of the photographic and photo-electric methods. In normal practice the plate is expected to give good results on ninth and tenth magnitude stars under less favourable conditions than this.

One cause of error in photographic photometry has still to be mentioned. This is the influence of light from the neighbouring stars or from the sky. The former not only introduce additional light (which can be roughly computed, and is usually negligible) but render the background uneven, so adding to the difficulty introduced by graininess in finding a part of the neighbouring plate with which to compare the image. Skylight may introduce errors which are difficult to estimate, for the effect produced by a faint illumination on an image may be much greater than its effect on the otherwise unexposed plate. The effects of exposing images already formed to long-continued faint illumination are in particular complicated and obscure, and the best way to avoid errors from skylight appears to be to shield all parts of the plate except those actually exposed to the light of the stars to be measured.

The irregularities in the "clear plate" measures of plates taken with the 24-inch reflector of the Royal Observatory, Edinburgh, which appeared to be due to the presence of stars too faint to be visible, prompted the writer to examine this effect further. A plate exposed on the Pleiades was taken and a search made for faint stars at the positions indicated by a long exposure photograph of the region. It was found that images giving below two divisions deflection, corresponding to an effective transparency of one per cent, were quite invisible, even when their positions were known. Images giving between two and four divisions deflection could not be found by eye, but could be distinguished when their exact positions had been found by the photometer. Such images as these could be located with considerable accuracy by the photometer and very fair estimates of magnitude made; and it seems likely that it would be of assistance in picking up faint images of satellites, &c., on plates of regions for which other similar negatives are available for comparison. It might indeed be possible after suitable comparisons to extend the range of stellar observations on the longest exposure negatives by a full magnitude by the use of the photometer.

V I.

PHOTOMETRIC METHODS

Terrestrial photometry is subject to difficulties arising in the first place in the realisation of a light source suitable for use as a standard; in the second place in the correlation of the luminous sensations of a particular eye with those of a standard eye or with some more ultimate units; and in the third in the comparison of two similar sources with each other. The last might be expected to give little trouble; but even here no great accuracy is secured; and when the photographic plate is substituted for the eye matters are made worse, for one of the most trusted measuring instruments - the rotating sector - is found to give results which are difficult to interpret. It is in the study of photographic plates, in fact, that the difficulty of reducing the intensity of a beam of light in a known ratio is most apparent; indeed, not many years ago it was stated by Dr. C. K. Mees that in investigating the deviations from the reciprocity law there is but one trustworthy method, namely: the inverse square law method.

In spite of this statement, a critical examination of the circumstances under which the inverse square law is applied do not suggest freedom from systematic error. A source of light which is assumed to radiate uniformly over a considerable solid angle is placed within an enclosure the walls of which are supposed perfectly black, and finally, the receiver is supposed to integrate the radiation falling upon it in a manner which is independent of the position of the source in the line joining source and receiver. The source, the walls and the receiver are all capable of introducing errors, and great precautions have to be taken if these are to be reduced below one per cent.

In the matters of an absolute standard and measurement in terms of that standard stellar photometry is no more advanced than terrestrial, and even in the comparison of the radiation of two stars of the same spectral class the difficulties have been considerable. For instance, in default of a method which could be considered free from systematic error, the Harvard North Polar Sequence was based on the mean of the results of a number of different methods, all known to be unsatisfactory in some respect or other.

The use of the photo-electric cell and thermopile, in place of the eye or photographic plate, makes possible a far higher order of accuracy than is reached in eye or photographic work in the measurement of intensity ratios. A method of comparing two similar sources of light to an accuracy of within one in one thousand was published in 1925 by Dr. N. R. Campbell and Mr. H. W. B. Gardiner.* A closely similar method has been in use at the Royal Observatory, Edinburgh, for many years for calibrating photo-electric cells; for although not previously published by him in relation

* *Journal of Scientific Instruments, 2, 1925, p. 177.*

to photometry the method is practically contained in Dr. Campbell's writings on absolute measurements,* which suggested it to the writer. Its freedom from troubles due to scattered or reflected light led to the adoption later of a somewhat similar method for the calibration of photographic plates, in spite of the fact that the conditions are much less favourable in this case than in the case of the photo-electric cell. To show the accuracy and certainty with which the latter may be calibrated the experimental details and results are given below:-

In testing the relation between the photo-electric current C and the illumination I it is simpler and more accurate to do so indirectly by examining the sensitiveness dC/dI for various illuminations. If a galvanometer is used for measuring the currents the main current through the cell may be annulled by that through a second cell lighted by the same lamp, so eliminating the irregularities due to lamp fluctuations. The sensitiveness itself is measured by the application of a small subsidiary illumination.

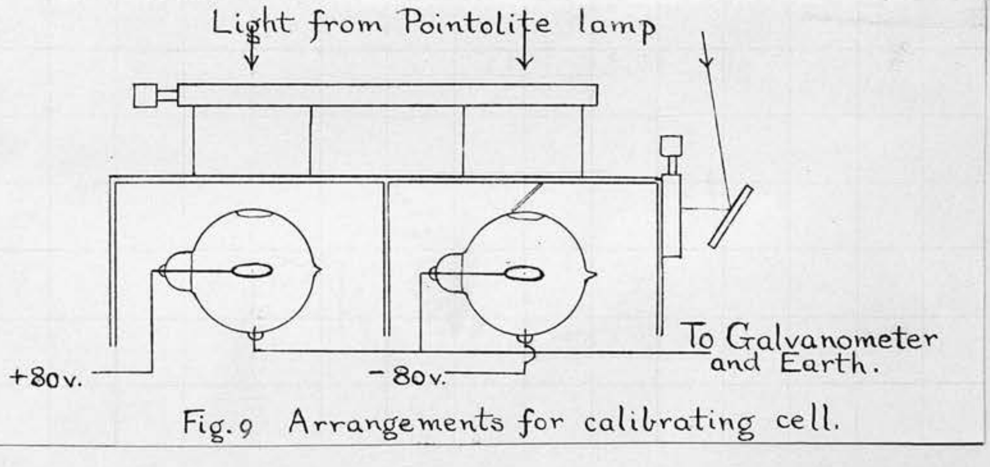


Fig. 9 Arrangements for calibrating cell.

A stereoscopic camera front was set up opposite a "Pointolite" lamp, and a reflector was added

* *See, e.g., "The principles of Electricity", London, (Jack) p. 37*

at the side as shown in figure 9 , with a separate shutter. The light passing through this shutter was partly reflected into the cell to be tested by a slip of glass in front of the cell window. Connections were made as shown to batteries and to a Broca galvanometer.

An example of the results is appended:-

	scale divs.
Current through cell when directly illuminated	1,230.
Added deflection produced by reflected light	24.0 ± 0.2
Deflection produced by reflected light alone	22.8 ± 0.2

The experimental error is then of the order of one in five thousand, though to keep the error down to these dimensions it is necessary to work slowly, resting the cell between exposures, and reading the deflection after a fixed interval of time from the commencement of the exposure. Using such precautions to avoid the effects mentioned in App. 4 , it was found for a certain cell that the error in assuming exact proportionality between illumination and deflection up to 500 divisions of the galvanometer (one division represented about 2×10^{-10} amperes) would nowhere exceed one per cent; and that by the use of a calibration curve it could be reduced below one in one thousand. It will be noticed that in the calibration method the presence of scattered light is of no importance, neither is there any possibility of effects such as interference or polarisation, either of which in certain circumstances might give rise to doubt as to the legitimacy of assuming that the effects of the different beams are additive. The calibrated cell was therefore used with perfect confidence for testing other photometric methods.

The object was to find a method which gave with simple and inexpensive apparatus results justifying the expectation that by its elaboration freedom from appreciable systematic error could be secured. It was found at once that without elaborate screens and curtains and a source of light of special construction there was no hope of reasonable accuracy from the inverse square law method. The polarising prism method was next tried, and gave good results up to angles of 60° from the position of maximum illumination. Beyond this the illumination indicated was in excess of the calculated amount, and there was besides some difficulty in getting rid of a slight prismatic shift of the light beam. No stress is laid on these results, which are presumably due to the use of imperfect prisms; but though the $\sin^2$ law has been verified in some cases, there have been other instances of doubt as to its accuracy.* If the only test of the perfection of the prisms is the manner in which they behave towards the $\sin^2$ law, they evidently cease to be of use in absolute measurements.

Another method tried was the use of a "neutral" glass whose absorption could be measured photo-electrically to almost any desired accuracy. The great difficulty found in working with so-called neutral screens or wedges was that of finding a material at once neutral and free from scattering particles, exact neutrality being especially difficult to secure in the ultra-violet. The presence of scattering particles, as in the Goldberg

** Bull & Cartwright, Jour. Sci. Inst. 1, 1923, p. 75. It may be noted that in testing the deviations from the reciprocity law by this method Dr. E. Kron placed no great confidence in the accuracy of the $\sin^2$ law; but refers to prisms tested by the inverse square law method. See Potsdam Publ. 22, 1913, Nr. 67.*

wedge, necessitates the calibration of the screen under the exact conditions of use. The Goldberg wedge is frequently placed immediately in front of the photographic plate, introducing errors from inter-reflections between plate and wedge which are difficult to estimate. A comparison of the value of Schwarzschild's index for a certain emulsion measured in this way with that found by the absolute method described later showed differences of two or three per cent; but it should be noticed that in these measures, as in all the writer's work with photographic plates, the area of plate covered was small, comparable, in fact, with the thickness of the screen used. Greater freedom from systematic error should be secured by the use of wedges and areas of plate large in comparison with the thickness of either; though under such circumstances errors due to irregular development and to variations of film thickness become important. The precautions to be taken in the use of the wedge under these conditions are described in a recently published booklet.*

One of the most promising methods tried was the rapidly rotating sector, the use of which, though condemned in the booklet just mentioned,+ has been found accurate under all the circumstances tried by workers at the U. S. Bureau of Standards and others. References, together with the writer's experiments on the subject, will be found in Appendix 3. Apart from the doubt regarding the validity of Talbot's law, considerable difficulty was found in obtaining uniform illumination over a stepped sector used close to the photographic

* *Dobson, Griffith and Harrison, "Photographic photometry." Oxford, Clarendon Press, 1926.*

\+ *Not content with the disparaging remarks given on pp.*

plate which it was the purpose of these experiments to calibrate. In its perfect neutrality throughout the spectrum and in the ease by which any desired ratio of absorption can be obtained, as well as in freedom from reflections, the rotating sector is unrivalled; and the objections which have been raised against it relate apparently to the error which enters in the use of a slowly rotating sector. Such a sector would scarcely ever be used in practice, owing to the difficulty of starting and stopping the exposure at the same position of the sector; the confusion appears to have arisen through the use of a sector as a means of measuring the duration of a continuous exposure, and from its use in sensitometry before the deviations from the reciprocity law were recognised.

The absolute method finally adopted was the addition method, the original form of which is described in appendix I. It is an attempt to realise the conditions for the addition of two equal illuminations and so produce an illumination equal to their sum; though in practice an allowance has to be made for the inequality of the original illuminations. In spite of the crude form of the original apparatus the method gave consistent results which appeared to be extremely reliable and free from systematic errors; the only loophole for such errors being apparently through the Eberhard effect, the images given by the separate and combined beams having slightly different areas. It was to test this point that the comparison of the

17 and 24 of this booklet, a reviewer again finds occasion to condemn the use of the rotating sector in connection with the mention of its use on p. 33 (See "Nature", 118, 1926, p. 689). In neither case is any evidence quoted for the opinions expressed.

addition and rotating sector methods described in appendix 3 were undertaken, and though the deviations found were of the same order as the experimental error, they pointed to systematic errors in both methods, of the order of one-half per cent. A new addition camera shown in section in figure 10, was therefore constructed in which any error arising from the Eberhard effect should be completely negligible. The effective light

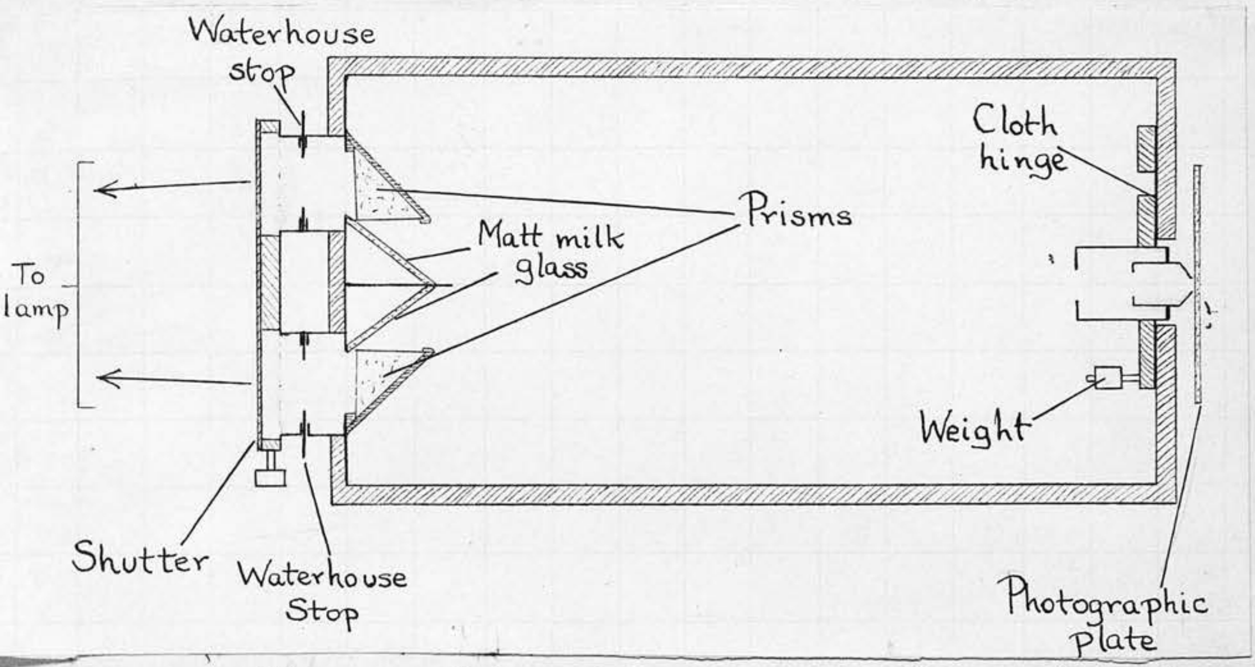


Fig. 10. Improved camera for addition method of photometry.

sources consist of two surfaces of ground white milk glass, separated by a thin blackened brass sheet, to give as little space as possible between them, and the diaphragm which limits the area of plate exposed to them is hinged in such a way as to fall into a position nearly touching the emulsion when the dark slide carrying the photographic plate is open. Any remaining systematic error in this camera is believed to be less than one in two thousand.

It might be supposed that a measurement of the area of the stops of this camera would be sufficient to give an accurate measure of the light passed. The difficulty here as in all similar methods, is to secure the required uniformity in the light beam. The misleading nature of the appearance of the surface illuminated by the beam to the eye has already been mentioned; and the only safe method of ensuring uniformity is to test the flux of light at different parts of the beam by a photo-electric cell. Good results may be expected from a light source consisting of a hole in the wall of a receptacle whitened within and containing a lamp or lamps; a somewhat similar arrangement may be used to integrate the light passing the measured stop and divert it on to the photographic plate.

The methods so far considered have been those applicable to the calibration of photographic plates or absorbing screens in the laboratory; and the experiments described were made with the object of calibrating the plates used in an investigation in stellar spectro-photometry.* The conditions in stellar photometry are peculiar in that the stars give perfectly uniform and parallel beams of light over the telescope objective, from which they converge to almost perfect point images. Of absolute methods which have been used on the telescope for determining intervals of magnitude those dependent on the use of polarising prisms have already been rejected as requiring independent tests. The others are the objective diaphragm, the objective grating, and the inverse square law method used by E. King+. It is surprising that the rotating sector has not been used in stellar photographic photometry,

* *R. A. Sampson, Monthly Notices, R. A. S., 83, 1923, p. 174; and 85, 1925, p. 212.*

+ *E. King, Harvard Annals, 59, 1912, p. 41.*

seeing that its use involves no appreciable disturbance of the distribution of the light from a star over its image; that it allows images of reduced and unreduced stars to be photographed at one exposure; and that it is a familiar adjunct in parallax work.

In the method used by E. King for work on the brighter stars the image of the star is used as a point source and the plate is placed at different distances from it until the deposit obtained as the result of a fixed exposure time has a standard density. Since the images of the stars heve unequal areas the larger will be more affected by the Eberhard effect that the smaller, leading to a systematic contraction of the magnitude scale. The resulting error cannot be more than a few hundredths of a magnitude over the range covered; however the method is not likely to be of lasting importance, for the bright stars covered by it are capable of being dealt with by photo-electric methods.

The objective grating has many advantages for stellar work. The images are all photographed at a single exposure, and are conveniently placed for identification. The magnitude interval between direct and diffracted image may be made as large as is desired by a suitable grating, and can be accurately computed from its dimensions. Its drawback is the difference of form between the direct and diffracted images. The effect of the change from a round to an elongated image may be deduced from the results found in chapters 3 to 5; from which it will be gathered that for denser images the diffracted images will be too large, while for the weaker they will be unduly weakened. The writer would expect that if the direct images were of the nature of the sharply focussed images discussed in chapter 3 the errors introduced into the results would be very considerable; the method has, however, been applied mainly to long focus instruments where the effects would not be so extreme. In the eye measurement of the images it is scarcely possible to say what form the error introduced by this change in the shape of the image will take.

The objective diaphragm has the same disadvantage as the grating, without any of its advantages. Its effect, however, will be in general to give the reduced images smaller than those taken with the full aperture, since unless the diaphragm is very small diffraction is seldom an important factor in determining image size. The effects of change of size are less important than those of the elongation produced by the grating, the images remaining symmetrical. It should be noticed that the method is practically restricted to a reflector owing to the fact that on a refractor the crown and flint components vary in thickness with distance from the centre, while in the shorter wavelengths the flint is the more strongly absorbent.

To sum up the contents of this chapter, then, it appears that there are laboratory methods by which light ratios may be determined to an accuracy of within one in two thousand; but that such methods are not directly applicable on the telescope, where on the whole the best absolute method appears to be the rapidly rotating sector, the systematic errors of which are of the order of one in two hundred when used with the photographic plate Where such errors are objectionable it may be possible to secure higher accuracy by the use of some intermediary calibrated by laboratory methods.

VII.

THE ILLUMINATION - EXPOSURE TIME RELATION FOR THE PHOTOGRAPHIC PLATE

The importance in photographic magnitude work of the questions discussed in the present chapter will be realised if an attempt is made to define the term "photographic magnitude" in a manner which shall lead to consistent results. There is one logical condition which should be fulfilled by a scale of photographic magnitudes - it may perhaps be best seen in the case of stars belonging to a single cluster. Suppose absorption in space to be completely negligible, then if the cluster is moved to a distance from the earth of for example ten times its original distance, the new magnitudes of its stars, irrespective of their spectral types, should be formed by adding a constant, in this case 5, to their original magnitudes.

It has been seen in chapters III and IV that if the magnitudes of the stars are derived on a plan which entails the images of the fainter stars being either smaller (in the case of focussed images) or less dense (with uniform images) than the brighter, this condition will definitely not be fulfilled - there will be a photographic Purkinje effect as foreseen by Hartmann as far back as 1899*. Strictly, as mentioned previously (p.4) the extent of our knowledge of the photographic plate does not justify the employment in absolute work

* *Astrophys. Jour.* 10, 1899, p. 229.

of images which differ in any respect whatever, whether it be size, density or exposure time.

The International scale of photographic magnitudes has been constructed mainly, though not entirely, on the principle of identical images. The brighter stars have been reduced to equality with the fainter by the use of wire gauze screens*, apparently the only means of doing so without affecting the distribution or colour of the light forming the image. Even here it appears that the brightest stars of the sequence have been exposed for considerably shorter times than the faintest, and this without experimental evidence that the colour sensitiveness of the plate is unaffected by altering the length of the exposure.+ The very use of the term "colour sensitiveness" as applied to the photographic plate is objectionable, for it is impossible to define it except in relation to some artificial set of circumstances. This has evidently been realised in framing the following "conclusion adopted" at the 1925 meeting of the International Astronomical Union: "*Il est important que les courbes de sensibilité spectrale pour l'ensemble: téléscopes, plaques, et filtres employés pour les observations photométriques soient déterminées et publiées.*" It is apparently left to the observer to decide what form the results should take, for no definition of the term "*courbe de sensibilité spectrale*" is given.

There seems little doubt that eventually it will be possible to manufacture photographic plates of fixed constitution and properties. Will it then be possible

**Objective diaphragms were also used, but only after it had been found by the aid of the wire gauze screen, that the error introduced was negligible.*

+In the discussion by F.H. Seares of the effects of varying the exposure time (M. Wilson Cont. 4, 1914, p. 298) the question of colour sensitiveness is omitted.

to define conditions under which photographic magnitudes should be measured and reduced which would eliminate the present inexact and arbitrary scheme of applying a colour correction to bring the results into harmony with the International System? The atmosphere certainly introduces difficulties; but no more, it appears, than might as in the case of refraction, be treated by suitable and determinate corrections. The telescopic absorption might be modified by the addition of a colour filter and so brought into line with a standard. The density of the imag might be fixed, say, to the value found in chapter IV to give the maximum absorption for a given flux of light, a value low enough to render the spreading of the image and the Eberhard effect of little consequence, so that the imag area would be immaterial. But if it were found necessary to fix a standard exposure time, it would presumably have to be made suitable for observations of the faintest stars, and would be so long as to prevent its adoption in general photometric work. It is therefore important to know, not only the form of the relation between the illumination and the exposure required to produce a given effect; but also the effect on that relation of the colour of the light.

It was found very early in the history of photographic photometry that the action of the light on the plate is not in accordance with the simple reciprocity law, effect $\propto$ illumination $\times$ exposure time; and the actual form of the relation has been the subject of much investigation, especially by astronomers. K. Schwarzschild in particular paid much attention to the question, and proposed the form

$$f(D) = It^{p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where D is the density, I the illumination, t the exposure time and p a constant over the region of exposure times used in astronomical work. He appears to have thought at one time that this quantity had the fixed value 0.86; but in the *Göttingen Aktinometrie* he used the value 0.76, remarking that the intermittency of the exposures might account for the small value.

In the modern work on the subject there are two investigations outstanding for their accuracy and range, namely, those by Dr. E. Kron at Potsdam Observatory,* and the as yet incomplete series of investigations by L. A. Jones, E. Huse and V. C. Hall at the Kodak Research Laboratory.+ Though in both cases results relating to the effect of wavelength are promised, they have so far not been forthcoming. Kron gives his densities in arbitrary units, from the values found for p they appear to be high. The results from the Kodak laboratories cover an enormous range of illuminations and exposure times, but they are confined to densities above 0.2 (diffuse). Below this density the most recent work is that by the writer, whose observations, over a much more restricted range of illuminations and exposure times, cover the density range from 0.01 to 1.5 (specular). Within this range the effect of the wavelength of the light has to some extent been dealt with. As regards density range these three independent investigations may be taken as complementary, Kron's results being applicable to dense (focussed) images,

* *Publ. der astrophys. Obs. zu Potsdam, 22, 1913, Nr. 67.*

\+ *Jour. Opt. Soc. Amer. 7, 1923, p. 1079; 11, 1925, p. 319; 12, 1926, pp. 321, 443.*

Jones, Huse and Hall's results to intermediate images, and the writer's to weak images. The writer has recently published a quantitative theory of the deviations from the reciprocity law, applicable to low densities;* the deduced formulae do not appear to be of practical interest except in so far as to act as a warning against extrapolation from results covering a more limited range of exposure times than is required. In the absence of any exact relation what is needed in photometry is a formula of simple type giving results of reasonable accuracy, so that in many cases the formula may be used as it stands, while in more exacting work it will still be of use, but subject to small corrections determined experimentally.

For this purpose formula (5) cannot be regarded as sufficiently accurate, unless the range of exposure time is small. The formula used by J. Halm at the Cape Observatory is

$$f(D) = \frac{2It}{i^{a} + i^{-a}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where $i = I/I_0$, a and I_0 being plate constants. Halm finds his observations to be satisfied by the value 0.25 for a. It is easily deduced from (6) that

$$\phi = \frac{1}{1 - a \tanh(a \log_e i)} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

ϕ being defined as in appendix 1, part I, p. (167). ϕ is then always between the limits $\frac{1}{1+a}$ and $\frac{1}{1-a}$, that is to say, between 0.80 and 1.33. These results are in good agreement with Kron's results, from which, in fact, they were deduced. The results from the Kodak Laboratory are also found to be in fair accordance with a formula of the type (6), but there are many exceptions, especially

* *Appendix 1, part V.*

with slow emulsions. Fig.11 shows some of the results obtained by Jones and Huse, compared with those given

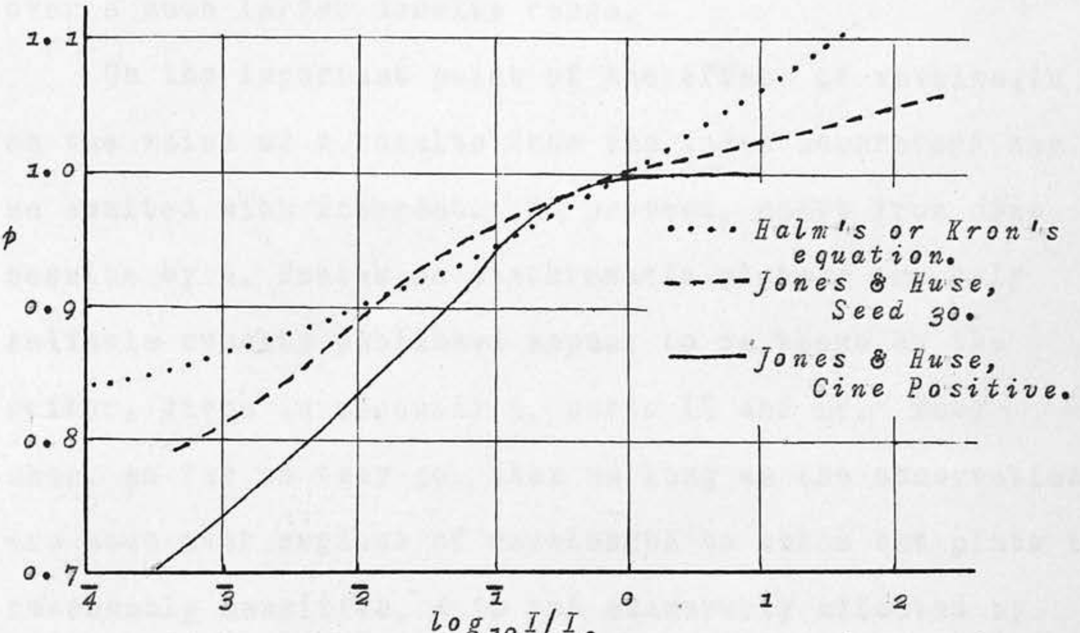
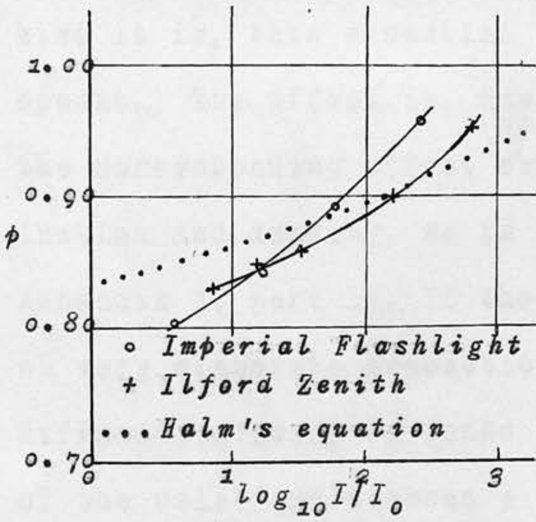


Fig. 11, Comparison of Jones and Huse's results with Halm's formula.

by the formula (7) with the value 0.25 for *a*.

With few exceptions the high density observations show that ϕ is, as (7) indicates, a function of the illumination *I*. At low densities the writer has shown that this ceases to be the case, instead, it is found that ϕ depends entirely on the duration of the exposure. This leads, as figs. 12*a*, and 12*b* show, to a difference between the relations of ϕ to *log I* according as the density or the exposure time is kept constant. The



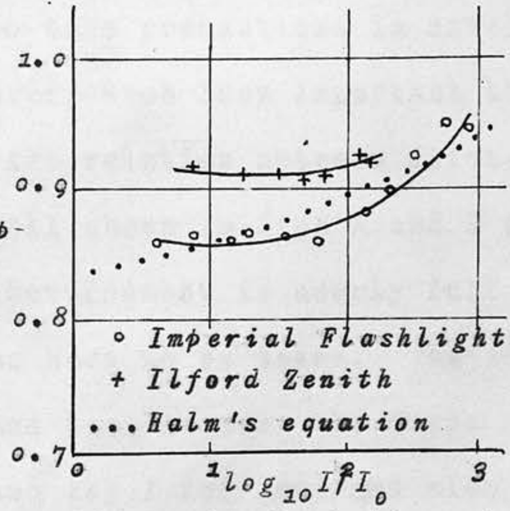


Fig. 12a. Variation of ϕ with log I, density constant

Fig. 12b. Variation of ϕ with log I, time constant.

figures show the results in normal cases - there are emulsions for which p depends only on the exposure time over a much larger density range.

On the important point of the effect of wavelength on the value of p results from the Kodak Laboratory may be awaited with interest. At present, apart from some results by A. Hnatek on panchromatic plates* the only reliable results published appear to be those by the writer, given in appendix 1, parts II and IV. They show, so far as they go, that so long as the observations are made over regions of wavelength to which the plate is reasonably sensitive, p is not measurably affected by wavelength. (See appendix IV, table 4.) The only case where an undoubted wavelength effect was noted is in the green with a panchromatic plate, in agreement with Hnatek's results.

Other factors which have been stated to influence the value of p are the extent to which the development is carried out, the speed of the emulsion and the age of the emulsion. The first effect is abundantly confirmed - p is higher the shorter the development. In the use of the relation between illumination and exposure time it is, then essential to take precautions in development. The effect is, however, much less important than the corresponding effect on the relation between illumination and density, as is well shown in figs 1 and 2 of Appendix 1, part IV. If the development is nearly full no very elaborate precautions have to be taken. The differences found by Jones and Huse between the forms of the relations between p and *log I* for fast and slow

*A. Hnatek, Zeits. wiss. Phot., 22, 1923, p. 177.

emulsions shows how dangerous it would be to follow a suggestion recently put forward* - namely, to assume that fast and slow plates give results which differ only in the units in which the illumination is to be expressed. The writer has noticed (Appendix 1, part II, p. (175)), a change of p with the age of the plate which might possibly be a source of error in photometry.

The very slight dependence of p on the wavelength makes the illumination - exposure time relation an alternative to the wire screen as a link between laboratory photometers and stellar observations. In one way it has a great advantage over the wire screen. In comparing the magnitudes of a bright star and a faint one, the exposures with the screen method are all equally long, the length being determined by the exposure required to give accurately measurable images of the faint star. With the illumination - exposure time relation the faint star alone needs a long exposure. The result in an indifferent climate especially is increased accuracy. An example of the application of the method is given in the next chapter.

* *H. S. Jones & J. Halm: Trans. Int. Astron. Union, 2, 1925, p. 91.*

V I I I .

MEASURES OF THE NORTH POLAR PHOTO-VISUAL SEQUENCE

At the time when the work described in the present chapter was started it was hoped to measure the photo-visual magnitudes of perhaps the first eighteen members of the North Polar Sequence, thereby fulfilling conclusion (2) of Commission (25) of the International Astronomical Union.* This intention has not been altogether abandoned, and in the normal course of events it would probably have been carried out more fully. The difficulty which arose through the lack of a guiding telescope to the 15-inch refractor of the Royal Observatory, Edinburgh, is one which would normally have been overcome; but at the present time, when important changes are being made in the instrumental equipment of the Observatory, expenditure of time and money on such an addition to the telescope was thought by Professor Sampson to be inadvisable. The measures have, however, been carried far enough to show the feasibility of the method employed - a method which in the writer's opinion offers the only sound alternative to the wire gauze screen as an intermediary between laboratory and stellar photometry.

The principle of the method will have been gathered from the remarks at the end of the preceeding chapter. The relation between illumination and exposure time for the plates and image density used is determined (by the laboratory method described on p. 58) and the exposure

* *Trans. Int. Astron. Union, 2, 1925, p. 238.*

time required for a definite effect on the plate is determined for each member of the sequence. There is an uncertainty introduced by the fact that usually the image density will not be uniform; but the error cannot be great. For one emulsion tried the density was immaterial, and though the batch of plates of the same emulsion next obtained did not give a similar result, it was found possible to use for this work two emulsions having their density effects opposite in sign.

In order to eliminate corrections for differential atmospheric absorption the stars were compared at times when their altitudes were equal. This did not appear likely to lengthen the work appreciably, since the eye-end of the telescope is constructed for visual work and gives a field only half a degree across, so that all but the faintest members of the sequence had to be treated individually. As it happened, the weather of the winter of 1925 - 26 proved less favourable than usual, with the result that no favourable opportunities occurred for observations satisfying this condition on certain of the brighter members of the sequence.

The telescope employed, a 15-inch visual refractor by Sir H. Grubb, was constructed in 1870. The mounting is of his usual form - the drive has a small periodic error which is inappreciable for stars so near the pole as are considered here. The telescope shows a very pronounced secondary spectrum and a slight prismatic effect which indicates a displacement of one of the lenses - the objective, however, has no means of adjusting the lenses either as a whole or singly. For photometry these are not serious defects. On the same mounting is a six-inch visual refractor by Dallmeyer, this was found to be too weakly attached to serve for guiding, and setting was done by a four-inch finder, by which the star could be brought within a millimetre of the centre of the field of the 15-inch refractor, which has a focal length of 183 inches. (4.65 metres)

In the absence of a guiding telescope all that could be done was to keep the clock running truly to sidereal time and to adjust the polar axis as closely as possible to the pole. The latter process was carried out by the method due to Blazko* which, though simple, does not seem to be so well known as it deserves. Trails are secured of three stars whose places are accurately known, the declination axis being clamped throughout, and the stars being conveniently one at upper culmination, one at lower culmination, and one six hours off the meridian. The three stars must evidently have their declinations within the range covered by the telescope field, and as it happens there are several stars suitably placed at just over three degrees from the pole. These are

	AR. h	AR. m	Dec. °	Dec. ′
Gr. 1004	6	19	86	45
Br. 1656	12	15	86	51
δ UMi.	17	56	86	37
Br. 2417	17	58	87	0
Br. 3147	23	28	86	54

The pair *δ UMi., Br. 2417,* is convenient as giving the scale of the negative in angular measure. No corrections for refraction are necessary, for the polar axis must be adjusted to the refracted pole, and differential refraction is negligible. The distances between the trails could be measured with an error comparable with

**Astr. Nach. 3452, 1897.*

the uncertainty of their positions, and the error in the position of the axis determined with far greater accuracy (approaching 0."1) than that possible in its adjustment. After finding the error the telescope was pointed to Polaris, and the wires of a filar micrometer adjusted to show the position of the star when the calculated correction to the polar axis should have been made.

The mounting of the telescope is supported by a box-shaped upper casting almost in contact with a fixed lower hollow casting to which it is secured by internal set screws. The arrangement entails a most awkward position for the person handling the screws, who must work in the cramped space within the castings. The arrangement of the screws is not geometrical, and some difficulty was found at the start in freeing the castings from each other, for they were found in actual contact. Originally, the axis was found to be 50" too low, and 70" too far to the West. After two days spent on adjustment this was reduced to 2" too high and 29" too far to the West. No certainty in the manipulation of the screws was attained, and attempts to improve on this result might have taken a considerable time. It was evident from the plates that the actual drift of the images was at least double the amount calculated from this error, a result probably due to flexure, the telescope tube being relatively weak.

On testing a number of emulsions for their speed in the green and yellow it was found that in this region plates of the most diverse H. and D. speeds gave almost identical results - thus, iso-chromatic plates of a speed 700 H. and D. gave no better results than

Screened or Rapid Chromatic plates with an H. & D. speed of only 225 to 300. Since fine grain is an advantage the slower plates were used, and Imperial NF. Ortho were selected. Since later it was found that a batch of Ilford Screened Chromatic plates, the colour sensitiveness of which resembled that of the Imperial NF. Ortho, showed no variation of ϕ with the density over a very large range, a change was made. The plates obtained, however, showed a considerable fall of ϕ as the density was increased, the first instance of this kind to come under the writer's notice.

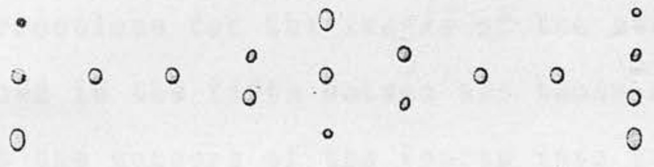
In order to avoid as far as possible the errors discussed on chapter I. the plates were obtained in a larger size than that necessary to cover the exposed area. The size was 5"×4", of which the area used was no more than 2" × ½". Development was done in an open dish rocked by hand; dish and developer having first been brought to the temperature 18° C. The NF. plates were developed for 5 minutes in Imperial Pyro-Soda, the Screened Chromatic for 6 minutes in Ilford Pyro-Soda.

The filter used was a Wratten K2, the absorption of which is stated by the makers to be as follows:-

Wavelength	6000	5200	5100	5000	4900	4800	4700	4600 Å
% transmitted	78	74	69	59	44	25	10	1.6

To avoid the possibility of trouble from dust or slight imperfections of the filter, the latter was placed at a distance of some two feet from the plate. Since the filter is cemented into plate glass the image is rendered more imperfect - this appears to be rather an advantage than a drawback, as it renders the density of the image more even. The plate was used about 2 mm. inside focus, for the same reason. The shutter used is a hand-operated sector shutter with wire release, of the type used for mounting in front of a camera lens. It covered a circle of about two inches in diameter, and was mounted close to the plate. With this shutter the uncertainty in the timing of the exposures should not be greater than one-tenth of a second. Moving parts above the plate are apt to drop dust on it; but a larger shutter placed above the filter proved too slow in action.

Stars 4 and 5 of the sequence were chosen as standards with which the other stars were to be compared, since the exposure they require is of the order 20 seconds, the smallest which does not involve appreciable errors in timing. Star 1 is too large and stars 2 and 3 too far from the pole to be useful as standards. In observing, stars are selected which have about the same altitude as the standard, and they are brought in turn to the centre of the field for exposure close to images of the comparison star. The arrangement of the images of plate 13, given as a specimen on p.78 are as follows:-



Star	4	8	9	5	4	5	9	8	4
Time	12 18 27	210	450	30 40	27 18 12	40 30	450	210	12 15 18 27

Any progressive errors of development, &c. along the plate are thus indistinguishable from the effects of variations of transparency of the sky, and should be eliminated over the whole plate.

The first step in the reductions is to express each star in terms of the log. exposure required by it to give the same result as unit exposure on the standard star. For this purpose a graduated series of exposures is given to the standard, enabling the photometer deflections to be converted (most conveniently by a separate graph drawn for each plate) into a scale of log. times of exposure to standard. By this graph all the deflections for all the images, those of the standard included, are converted to log. times, and a scheme of corrections is deduced from the differences between the converted and actual log. times of the standard, taking into account both the order and the density of the image. To make the process clear the reductions for plates 5 and 13 are given below. The first column gives the number of the star in the North Polar Sequence, the second the exposure time in seconds, the third the photometer reading of the corresponding image, the background being brought to 100.0. The graph by which the readings are converted to log. times is deduced from all the observations of the standard star on the plate, which may, as in the case of plate 5, show a considerable drift due to change in the atmospheric transparency. The corrections for the images of the standard star contained in the fifth column are those necessary to convert the numbers of the fourth into $\log_{10}$ (16), (24), or (36) as the case may be, the intermediate corrections are deduced from these, and applied to give the numbers in the sixth column.

PLATE	Star	Exposure seconds	Photometer	Do. converted to log t. scale	Corrns.	Equiv. log t on (4)
5	4	16	46.2	1.209	5	
1926	"	24	28.2	1.375	5	
JAN 4	"	36	16.1	1.541	15	
SID.	2s	40	28.2	1.374	15	1.389
TIME	3r	120	37.8	1.278	22	1.300
2½ h.	4	16	52.2	1.168	36	
- 3¼ h.	"	24	31.8	1.334	46	
NF.	"	36	17.4	1.519	36	
4915	7.	120	35.5	1.298	51	1.349
FOCUS	3r	120	38.9	1.268	61	1.329
11.5	2s	40	32.1	1.332	71	1.403
	4	16	60.2	1.125	79	
	"	24	35.8	1.295	85	
	"	36	17.8	1.512	44	

PLATE	Star	Exposure (seconds)	Photometer	Do. converted to log t scale	Corrns.	Equiv. log t on (4)
13.	4	12	61.2	1.075	4	
1926 FEB.		18	45.1	1.259	4	
27-28		27	30.0	1.456	25	
SID.	8	210	42.2	1.298	0	1.298
TIME	9	450	45.2	1.258	0	1.258
10h. - 11h.	5	40	43.8	1.277	0	1.277
S.C.		30	52.0	1.177	2	1.175
6000F.	4	12	60.0	1.088	9	
FOCUS		18	45.8	1.251	4	
12.0		27	31.8	1.434	3	
BRIGHT	5	30	53.0	1.166	0	1.166
MOON.		40	42.8	1.290	2	1.292
LOW CLOUD	9	450	43.1	1.286	3	1.289
AT FINISH	8	210	44.8	1.263	6	1.269
	4	12	61.6	1.071	1	
		15	52.2	1.175	8	
		18	46.1	1.249	6	
		27	35.0	1.393	38	

The laboratory measures of the quantity ϕ, defined in Appendix 1, part I, p.(167) were made as described on p.(170) of that appendix, using the camera described in chapter VI (p. 59), together with a gas-filled lamp and a K2 filter. Since the value of ϕ was found to be independent of wavelength within wide limits the precautions to obtain comparable colour of light in the stellar and laboratory work need not be excessive. The comparatively small effect of the density on ϕ, unless the density is high, makes it possible to use the values obtained from uniform images for the stellar images, which, though not uniform, were nowhere dense. The effect of density in the two cases is seen in the last five rows of table I, and the first five rows of table II. The laboratory measures were co-terminous with the stellar, and the plates were selected at random from the boxes used for the stellar work. The only circumstance not exactly parallel in stellar and laboratory work was the temperature at the time of exposure - advantage was taken of a replacement in the Observatory heating apparatus to test the question of the dependence of ϕ on the temperature, without finding any marked effect.

The resulting values of ϕ are given in the following tables:-

TABLE I. - Values of p for Imperial NF. Ortho. Plates, Batch 4915.

Density	*Exposure time (mean) in seconds*	*Values of p found*					*Mean p.*
1.10	12.6	0.990,0.996					0.993
1.11	24.4	0.995,					0.995
1.11	54.7	0.982,0.973					0.978
1.16	123.	0.944					0.944
1.11	243.	0.888,0.898					0.893
0.014	5.5	0.908	0.911	0.977	0.956		0.938
0.017	12.6	.914	.932	.940	.905		0.923
0.020	24.6	.884	.931	.928	.939		0.921
0.020	109.	.861	.889	.876	.898		0.881
0.016	480.	.845	.871	.866	.861		0.861
0.37	11.2	0.922	0.959	0.980	0.956	0.961	0.956
0.54	24.8	.950	.957	.919	.938	.932	0.939
0.56	51.2	.918	.938	.942	.908		0.926
0.51	110.	.886	.903	.885			0.891
0.61	239	.886					0.886
0.56	573	.853	.855	.872			0.860
2.20	42.3	1.007	0.979	0.967	0.983	0.983	0.984
1.41	41.6	0.950	.942	.954	.956	.941	0.949
0.73	41.2	0.929	.923	.923	.920	.924	0.924
0.11	41.2	.928	.922	.938	.924	.914	0.925
0.016	40.8	.935	.876	.916	.916	.	0.911

TABLE II. - Values of p for Ilford Screened Chromatic Plates, Batch 6000F.

Density	Exposure time (mean) in seconds	Values of p found					Mean p.
1.51	40.0	0.847	0.853	0.852	0.866		0.855
1.19	40.4	.887	.882	.867	.867	0.877	0.876
0.52	40.4	.885	.870	.885	.910		0.888
0.09	41.0	.921	.911	.903	.881		0.904
0.015	41.1	.914	.910	.894	.915	.920	0.911
0.24	6.9	0.915	0.972	0.914	0.934	0.933	0.934
0.33	30.0	.895	.898	.891	.896	.891	0.894
0.32	134.	.852	.854				0.853
0.24	290.	.846					0.846
0.21	615.	.844	.847				0.846
0.037	60.0	0.881					0.881
0.26	7.4	0.944	0.975	0.920			0.946
0.28	30.6	0.888	.922	.920			0.910
0.26	134.	0.877	.873	.861			0.870
0.18	615.	0.873	.853				0.863
0.24	7.4	0.955	0.923	0.923	0.952		0.938
0.28	30.6	0.904	.891	.899			0.898
0.24	134.	0.894	.868	.875			0.879
0.17	615.	0.846	.840	.843			0.843

The formula found to satisfy these observations is

$$m = m_0 + ax - b \log_e \cosh x$$

where $x = \log_{10} t/t_0$. On differentiation this gives

$$2.5\,\phi = a - b \tanh x$$

For the constants the values $a = 2.275$; $b = 0.167$ were adopted for both emulsions. For $\log_{10} t_0$ the value 1.87 was taken for the NF. Ortho plates, and 1.30 for the Screened Chromatic plates. The following shows the agreement obtained:-

TABLE III. Comparison of computed and observed values of ϕ.

Values of ϕ for NF. Ortho plates, density 0.5			Values of ϕ for Screened Chromatic plates, density 0.3		
Computed	Observed (table I)	Diff.	Computed	Observed (table II, means)	Diff.
.955	.956	-.001	.938	.939	-.001
.940	.939	+.001			
.928	.925	+.003	.898	.901	-.003
.921	.926	-.005	.865	.867	-.002
.899	.891	+.008			
.879	.886	-.007	.851	.851	.000
.863	.860	+.003			

Of the twenty plates which comprise the stellar material ten contain the observations on stars requiring upwards of ten minutes exposure. Those images which were given exposures exceeding twenty minutes are obviously elongated, and it was thought advisable to reject all over ten minutes. Three other plates were rejected for wide and irregular variations, presumably in atmospheric transparency. The observations on the remaining seven plates are given below:-

TABLE IV. Observations of stars between 4th and 9th magnitudes.

Star	log. exp. time	Comp. star	Equiv. log. time	Emulsion	No. of exposures star	No. of exposures comp. star	Difference of magnitude Obsd.	Difference of magnitude Intl.
2s	1.602	4	1.396	NF.	2		0.481	0.46
3r	2.079	"	1.315	"	2	9	1.759	1.73
7	2.079	"	1.349	"	1		1.679	1.71
8	2.176	5	1.470	NF.	2	9	1.611	1.68
9	2.477	"	1.450	"	2		2.321	2.38
7	1.872	5	1.451	NF.	2	12	0.972	1.10
10	2.556	"	1.420	"	2		2.564	2.61
1	0.699	4	1.305	SC.	6		-1.408	-1.47
8	2.255	"	1.214	"	1	9	2.304	2.29
9	2.556	"	1.182	"	2		3.021	2.99
4r	2.380	"	1.313	"	1		2.344	2.43
1	0.699	4	1.308	SC.	6		-1.414	-1.47
8	2.255	"	1.219	"	1	9	2.290	2.29
9	2.556	"	1.153	"	2		3.087	2.99
4r	2.431	"	1.376	"	1		2.310	2.43
8	2.322	4	1.284	SC.	2		2.285	2.29
9	2.653	"	1.274	"	2	10	3.016	2.99
5	1.477	"	1.171	"	2		0.695	0.61
5	1.602	"	1.285	"	2		0.713	0.61
2r	1.301	5	1.319	SC.	6	10	-0.041	-0.13
2r	1.301	"	1.333	"	5		-0.073	-0.13

In view of the small number of observations a solution by least squares may be replaced by the following simple proceedure. The observations may be reduced to one scale by the use of the value for the magnitude interval between stars 4 and 5. For this interval the following observations are available in the above table:-

		Wt.	Emulsion
Through star 7	0.707	1	NF.
" " 8	0.682	2	NF & SC.
" " 9	0.720	2	NF & SC.
Direct	0.704	3	SC.
Weighted mean ..	0.703		

Taking the magnitude of the star 4 as 5.800, the resulting scale is as follows:-

Star	Magnitude Obs^d.	Inter^l.	Star	Magnitude Obs^d.	Inter^l.
1	4.389	4.37			
4	(5.800)	5.84			
5	6.503	6.45	2s	6.281	6.30
7	7.477	7.55	2r	6.446	6.32
8	8.098	8.13	3r	7.579	7.57
9	8.837	8.83	4r	8.127	8.27
10	9.067	9.06			

Except in the case of the red stars the agreement is satisfactory. It speaks well for the method to find no sign of any systematic difference between the results for the two very different emulsions used, in spite of the fact that only one constant in the reduction formula had different values for the two emulsions. With a guiding telescope there would be no difficulty in extending the observations to the 12th magnitude or even fainter, though not many nights would be found suitable for the exposures required. In addition to a guiding telescope some form of rapid motion in right ascension which could be worked from its eye-end is necessary for rapid work near the pole - the absence of such a motion necessitated a series of trials at each setting and made the work slow and laborious.

VI. *THE INFLUENCE OF WAVELENGTH ON DENSITY GROWTH*

In the year 1889, that is to say, in the year previous to the appearance of the Hurter and Driffield system of sensitometry, Sir William Abney was expressing the growth of density of a photographic plate by the formula

$$D = K(\log E/E_0)^2 \quad \ldots\ldots\ldots\ldots \quad (1)$$

D being the density and E the exposure (the product of illumination and duration) and K and E_0 plate constants. Although the work of Hurter and Driffield established the use of the formula

$$D = \gamma \log E/i \quad \ldots\ldots\ldots\ldots \quad (2)$$

in sensitometry, it was in no way because of its accuracy as a representation of the actual growth of density with exposure. In the low density region, in fact, formula (2) fails altogether, and from the point of view of photometry, where such considerations as the preservation of the contrasts in the objects photographed do not enter and accuracy is important, formula (1) is in general to be preferred.

Since attention was recently drawn* to the fact that no theoretical foundation has been proposed for Abney's formula, the following ideas, leading *inter alia* to Abney's formula, may be of interest. As will be seen, no attempt is made at a rigorous treatment; any such attempt appears to be doomed to result in formulae so

* *F. E. Ross, "The Physics of the Developed Photographic Image, Eastman Kodak Co., New York, 1924, p. 49*

involved and depending on so many factors as to be useless in practice.

Those grains of a photographic emulsion which lie beneath the surface will receive only that portion of the light which has not been absorbed by the surface grains, hence during an exposure to a uniform illumination we may consider the emulsion divided into a number of strata, the grains of any one stratum being equally illuminated. Consider now the result of supposing that each stratum separately obeys the formula (2) from the point at which the effect commences up to the point at which all the grains are developable, when no further action is supposed to take place. The result is shown in fig. 1, by the broken line A. To see the effect

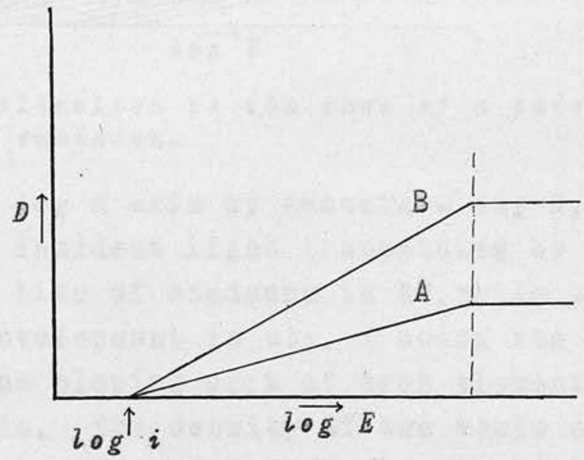


Fig. 1. Effect of combining two similar strata.

of increasing the number of grains in the stratum, suppose the number to be doubled. The effect will be the same as if two similar strata were fused into one; D will be doubled throughout, and the resulting stratum will obey the same law as the first, the value of γ and the maximum density being doubled, while the projection of the sloping portion of the curve on the $log\ E$ axis remains unaffected. (Fig. 1, B.) It appears, then, that in a single stratum D_{max}, the upper limit of density, is proportional to y .

Suppose next that the emulsion is made up of a number n of exactly similar strata, superposed one on another, and that during the exposure each transmits a fraction θ of the light incident upon it. The elementary curves of the type A (fig. 1) will then be displaced one after the

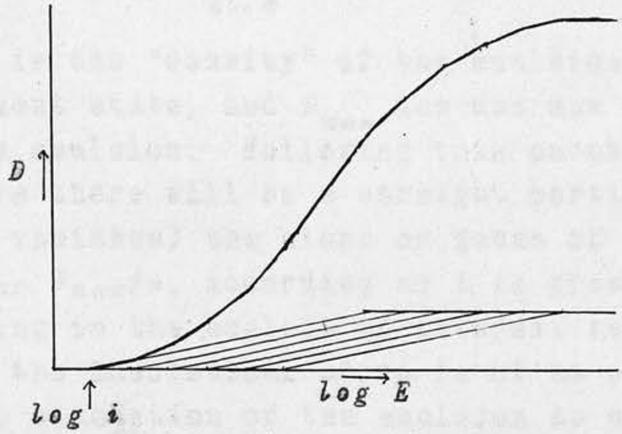


Fig. 2. Application to the case of a thick emulsion.

other along the $\log E$ axis by amounts $-\log\theta$, and the fraction of the incident light transmitted by the whole emulsion at the time of exposure is θ^n, while the maximum density after development is nly, l being the constant projection of the sloping part of each elementary curve on the $\log E$ axis. The density of the whole emulsion corresponding to a given value of E is the sum of the densities of its components for that exposure (fig, 2) hence at the point where the $(r+1)$th stratum begins to be affected the density is

$$D = -\frac{r(r+1)}{2} y \log\theta$$

If then n is supposed increased without limit, under such conditions as to leave the maximum density and the fraction of light transmitted by the emulsion during exposure finite, the equation for the initial portion of the "characteristic curve" of the complete emulsion will be will be, putting $\log E = \log i - r \log \theta$

$$D = -\frac{y}{2.\log\theta}(\log E/i)^2$$

$$= -\frac{nly}{2l.\log\theta^n}(\log E/i)^2$$

$$= \frac{D_{max}}{2l.\phi}(\log E/i)^2$$

where ϕ is the "density" of the emulsion in its pre-development state, and D_{max} the maximum density of the complete emulsion. Following this parabolic portion of the curve there will be a straight portion (unless $l = \phi$, when it vanishes) the slope or gamma of which is either D_{max}/l or D_{max}/ϕ, according as l is greater or less than ϕ. Owing to the neglect of reversal the remaining portion of the theoretical curve is of no practical interest.

The conception of the emulsion as made up of similar strata each separately obeying the formula (2) leads then to the expression $D_{max}/2l\phi$ for the constant K of Abney's formula (1), and to one or other of the expressions D_{max}/l, D_{max}/ϕ, for the value of gamma. The density ϕ of the emulsion before development may of course be measured, and the value of l deduced by noticing that the projection on the $\log E$ axis of that part of the curve lying between the vertex of the parabolic initial portion and a point half-way up the straight part is the arithmetic mean of l and ϕ. The results are actually of the right order - numerical agreement, however, is scarcely to be looked for. The value of the results lies in what they suggest is the influence of the quantity ϕ on density growth, for it appears to be through this quantity that any effect of wavelength on the shape of the characteristic curve enters.

It is suggested, then, rather than shown, that the quantity K in Abney's formula is inversely proportional to ϕ, the density of the emulsion before development, and depends therefore on the wavelength; that some emulsions have gammas independent of the wavelength while for others the gammas are inversely proportional to this same quantity ϕ, and that the distinction between the two is largely a question of thickness. It is quite possible - perhaps probable - that over one region of wavelength an emulsion may be in the former class while in another region it falls into the latter. It seems worthy of consideration whether the apparently contradictory results which have been obtained experimentally for the effect of wavelength on gamma may be reconciled by including emulsion thickness as an important factor in the problem.

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(APPENDIX 3)

A CONVENIENT PHOTO-ELECTRIC PHOTOMETER AND DENSITOMETER.

By E. A. BAKER, B.Sc., Royal Observatory, Edinburgh.

[*MS. received*, 30*th May*, 1924.]

ABSTRACT. A short period electrometer is described which has an exceptionally long scale and gives readings proportional to the ratio of two potentials. It is applied in the construction of a photo-electric photometer and a photo-electric densitometer, both instruments being independent of battery variations.

The sensitivity and quickness of action of a photo-electric cell as a photometer are best realized by using it in connection with a short-period electrometer and high resistance. Most arrangements of this kind require a very steady high potential battery, but by using the electrometer described below the readings are unaffected by small variations in the high potential, which may therefore be derived from small dry cells or from a dynamo. The electrometer, which was developed from a double plate electrometer shown to the author by Dr R. T. Beatty*, has the further advantage of an exceptionally long scale of 500 divisions or more.

Two vertical brass plates, one earthed, the other insulated, are mounted on a horizontal slide movable by a fine-pitched screw with micrometer head. A Wollaston wire is fixed at its upper end to an insulated support and hangs between the plates, its lower end being observed in a microscope with cross-wires. When wire and plates are all earthed the cross-wires are adjusted to coincide with the image of the wire. In use the wire is charged to a potential of the order of 100 volts, the insulated plate is charged to the potential to be measured, and the micrometer screw is turned until the image is again on the cross-wires. The micrometer reading is evidently independent of the elastic constants of the wire and of gravity and therefore depends only on the distance between the plates, and on the potentials of the charged plate and wire. It follows from the physical dimensions of these

* See *Phil. Mag.* **14** (1907) 606.

quantities that it is proportional to the plate distance and depends only on the ratio of th two potentials, to which it is in fact also proportional.

The plate distance is conveniently made variable, as in Fig. 1. Here the plates A, A between which the wire B hangs, are carried by amber insulators C, C set in the lugs of nuts D, D with right- and left-hand threads on a spindle E which can be turned by inserting a screw-driver in the hole on the right. The spindle is pressed by a spring F against the micrometer screw G on the left; a loose pin H between them prevents the screw from turning the spindle. A weak spring K between the nuts takes up backlash and a guide rod L below the spindle keeps the nuts upright. The author uses a platinum wire 4 cm. long and $7\,\mu$ thi between plates 10 mm. apart: a 1 inch objective is sufficient to realize the full accuracy of a micrometer screw reading by estimation to $1\,\mu$, and, since the wire follows almost im mediately any motion of the screw, readings to this accuracy can be made in two or thr seconds. The wire is stable if it is kept near its cross-wires by the screw and has a consider able range of stability on each side, but where large sudden deflections are likely to occ

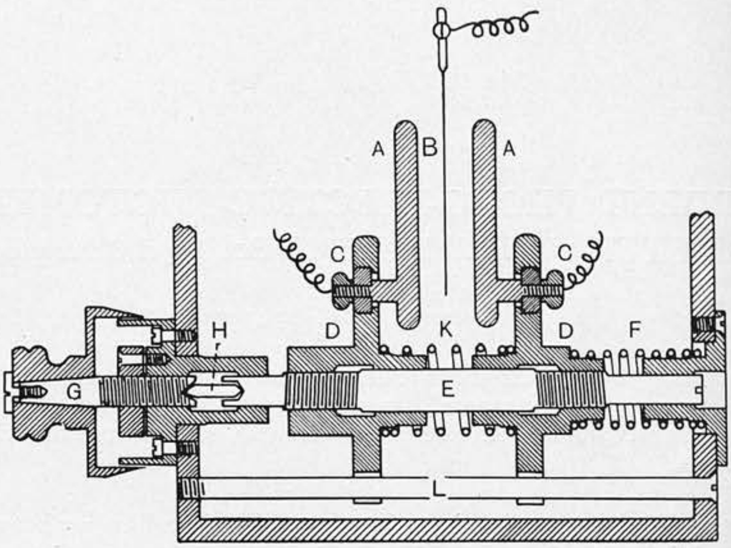

Fig. 1. Vertical Section of Electrometer

it is preferable to fix both ends of the wire and to observe its middle point as in a thr electrometer.

The instrument is shown connected to a photo-electric cell in Fig. 2. The cell A, of t cylindrical type described by Dr Beatty*, has its cathode surface connected to a battery a to the Wollaston wire B of the electrometer. Its anode C is connected by the flexible wire to the insulated plate and also through a high resistance R to earth; the guard ring G of t cell and the positive pole of the battery are also earthed. The readings, depending on t ratio of the plate and wire potentials, will be independent of the value of the battery potent if the photo-electric cell obeys Ohm's law. Though this is not true in general, there is f every gas-filled cell a range of voltage over which the law is very approximately obeye and by adjusting the pressure of gas in the cell the centre of this range may be fixed at a point from a few volts up to several hundred volts.

The sensitivity of the instrument depends on the value of the high resistance. For w where the highest sensitivity was not required a resistance of about 10^9 ohms, consisting a fine lead pencil line on ebonite, was used. This type of high resistance is easily made a

* *Phil. Mag.* **33** (1917) 52.

appears to be reliable. For densitometry and for absorption measurements, however, the high resistance is best replaced by a second photo-electric cell illuminated directly by the light source used for the measures, since variations of the light source are then eliminated*. It is preferable to avoid very low potentials over this second cell since for potentials below 1 volt its voltage-current relation is complicated and the final calibration is troublesome. The difficulty is avoided by illuminating the high potential cell, not only through the absorbing

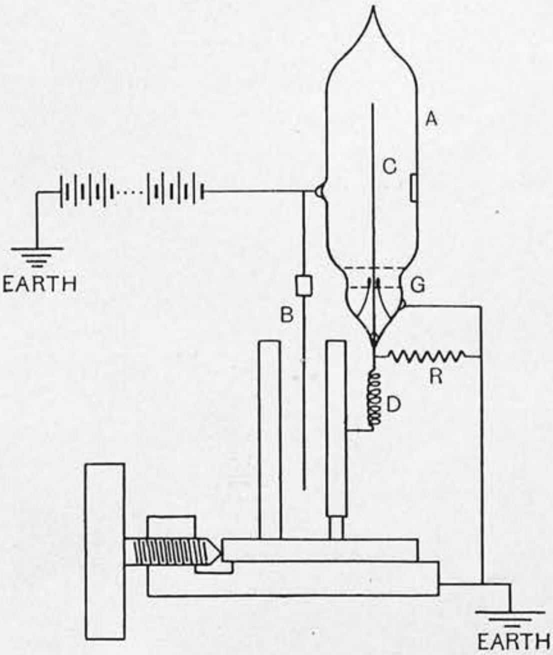

Fig. 2. Electrometer connected to Photo-electric Cell

medium, but directly through a second window by the same light source, and by charging the electrometer plate which was formerly earthed to, say, 5 volts by a potential divider from the battery. A densitometer of this construction may be made sufficiently independent of the battery potential for use on a direct current supply from a dynamo, while the sensitivity can be varied within wide limits by adjusting the illumination of the low potential cell. For example, with a 100–110 volt supply and a Pointolite lamp, it is possible to obtain density readings in two or three seconds using an area of a photographic plate not more than 1 mm. by ·02 mm.

* Cf. P. P. Koch, *Ann. d. Phys.* **39** (1912) 705.

APPENDIX 4. - THE KOCH PHOTOMETER OF THE ROYAL OBSERVATORY, EDINBURGH.

In view of the rapidly spreading use of electrical methods of measuring photographed spectra the following description of the Koch pattern microphotometer* set up a few years ago at the Royal Observatory, Edinburgh, may be of use to those who contemplate the installation of a device of that nature. The requirements in different lines of work are so varied that the instruments on the market are not always satisfactory for a particular purpose. The one to be described, for instance, was designed for photometric work on stellar spectra and not for great accuracy in measurement of positions of lines; and seeing that a large number of spectra had to be dealt with, each of considerable length, it was desirable that the instrument should run with the minimum of attention. In this respect the Koch photometer, which is independent of fluctuations in the light source, has a manifest advantage over thermo-electric recording instruments.

The complete instrument (plate 1) consists of the following separate parts:-

1. The measuring machine, carrying the plate, slit and optical parts.
2. The electrometer and cells.
3. The lamp.
4. The recording camera.

together with auxiliary apparatus - driving motor, batteries, switchboard, &c.

The measuring machine, a superseded instrument from the Observatory store, is a well-designed machine for the date (about 1880) of its manufacture; but the pitch of the screw, which has 50 threads to the inch, is too

*P.P.Koch, *Ann. d. Physik*, 39, 1912, p. 705.

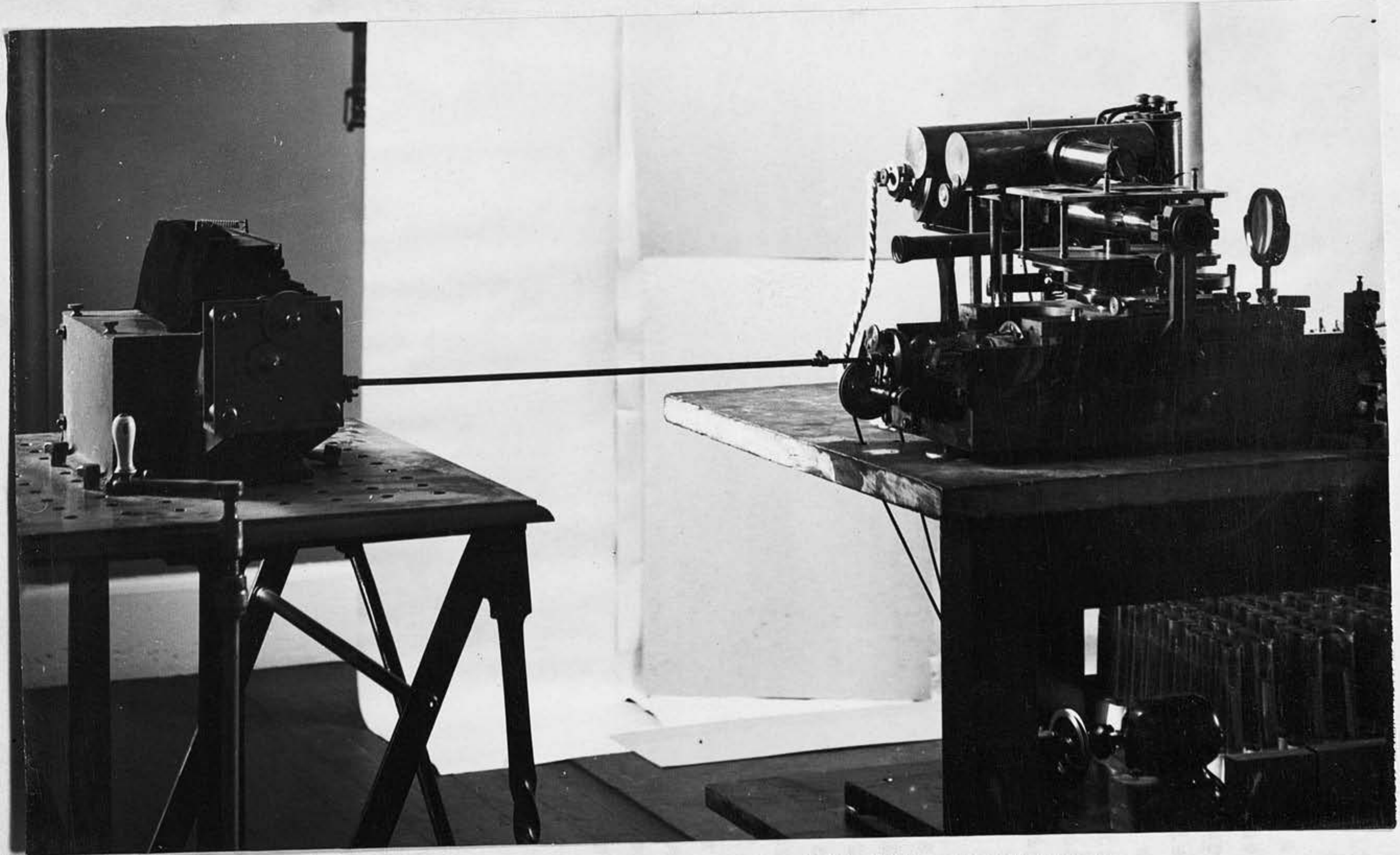

PLATE I. - GENERAL VIEW OF COMPLETE INSTRUMENT.

small for the weight of the moving parts, which, like the remainder of the machine, are very heavily made. The moving carriage, which weighs about 25 lbs., has a feature which is almost essential for rapid work on narrow spectra, namely, a well made rotating table with clamp and slow motion, above which the plate table proper is carried on a cross slide. The vertical plane in which the axis of rotation moves passes through the axis of the beam of light by which the deposit is measured, as shown in the end-on view (plate 2) and for a certain position of the carriage the two axes coincide. In this position of the carriage one end of the spectrum is brought over the beam of light by the centering screws of the cross-slide; when evidently a small rotation of the table will produce no appreciable effect on the beam of light. The carriage is then moved until the other end of the spectrum is near the beam, when it is brought central by rotating the table. The movements of the electrometer thread furnish a very precise guide to the centering. In most laboratory work on spectra exact adjustments of this kind would be unnecessary; but in this instance the spectra are little more than half a millimeter wide, while for photometry the light beam must be wholly within the spectrum, and for the maximum resolving power must cover as much of the width as possible.

The chief error to be feared in designing the optical system is the effect of light scattered from regions of the plate outside the direct course of the beam. An error of this kind was present in the original Hartmann microphotometer,* and to avoid it as much of the plate as possible should be shielded from light. The incident beam was therefore made to pass through a narrow slit

* *Schwarzschild and Villiger, Ap. J. 23, 1906, p. 286; Tugman, Ap. J. 42, 1915, p. 326.*

placed below and almost in contact with the emulsion of the plate, the distance between the two being less than one-tenth of a millimeter. This necessitated very careful construction of the plate table in order to preserve this distance over the whole of the run of the screw, which is five inches; but was found to give such resolving power as to make an enlarged image and second slit quite unnecessary. The instrument, in fact, is capable of resolving the grain of any but wet plates, even a process plate showing considerable grain when the slit is reduced to 0.5mm × 0.005mm, an aperture at which the photometer is not unduly sluggish. The slit is firmly supported on castings fixed to the base, and is mounted on a rotating base to allow it to be adjusted to the varying inclinations of the lines found in objective prism spectra. A little practice enables this inclination to be estimated at a glance to within two degrees, an accuracy which is ample with such narrow spectra.

To do a variety of work in an efficient manner one optical system is not sufficient. Narrow spectra require a concentrated beam of light and a short slit; whereas spectra taken with a concave grating are naturally wide, owing to the astigmatism of the grating, and the greatest resolving power is obtained with a long slit. The arrangement used for stellar spectra is shown in fig. 1 ; for solar spectra the lens of the condenser is replaced by a cylindrical lens having its focal lines, one at the plate, the other at the cell window. By this means a slit 10mm. long can be used. The lamp is a 100 c.p. Pointolite (fixed focus type), which beside furnishing the measuring beams acts as a projection lamp for the electrometer thread, using a concave mirror in place of a condenser. The arrangement, apart from its convenience and economy, has the advantage of leaving an indication on the record if any serious alteration in the strength of the light occurs.

The electrometer is the Cambridge Instrument Co's "Laby" pattern, which though now apparently obsolete has some advantages for this purpose, for it is very rigid, and the adjustments of the microscope and thread tension leave nothing to be desired. Its fault lay in the casing. The microscope and the lever supporting the lower end of the thread pass through holes in the case wide enough to allow of side motion, it was consequently impossible to keep the interior dry and free from draughts. The case was therefore re-designed, the holes being replaced by short tubes connected to the moving parts by thin pliable rubber tubing. (Suitable tubing is sold in the form of grips for the handles of golf clubs) The quartz insulators of the thread were replaced by amber, and the silvered quartz fibres abandoned in favour of the more durable platinum Wollaston wire, 7μ thick. The projection system consists of a Leitz No. 3 objective and in place of an eyepiece, a Barlow (achromatic concave) lens, the combination covering a field eight inches across at a distance of two feet from the lens. The sensitiveness is limited by the lack of constancy of the battery. No appreciable shift of the zero must occur during the ten minutes or so occupied by a record - this limits the sensitiveness used to about 20 mm. per volt.

The speed of working with the Koch photometer is largely influenced by the capacity of the insulated system - in this respect the writer has been able to improve considerably on the figures quoted by Koch (35 cm.) and Goos (60 cm.) by constructing cells of a cylindrical type having a very low capacity, and by using, as shown in fig.2, the central electrode as the cathode of the low potential cell. The electrical connections are in other ways similar to those of Koch's original instrument. The capacity was tested by two methods, one involving direct comparison with an insulated ball, the other reducing a measured light current in a known ratio until it was small enough to allow the rate of charge of the thread to be measured. The result in both cases was in the neighbourhood of 15 cm. The cells are of potassium, sensitised and filled with Helium, which gives a sparking potential of about 155 volts.

The bromide paper camera, another piece of apparatus discarded and in store, takes continuous rolls of paper 30 feet long and of any width up to nine inches. Its clock was replaced by a gear having two wheels detachable and interchangeable. Two sets of wheels give therefore four speeds, which are arranged to give linear magnifications of about 2.5, 5, 10, and 20 times the original spectrum. The wheels must be accurately cut and fitted, with diagonal teeth, or the accuracy of the drive is much impaired. A worm drive is preferable; but the accuracy obtained from the records is fully as good as the accuracy of the present screw warrants, that is to say, it is within a few μ on the original negative, when the highest magnification is used. The drive, as plate I shows, is taken directly from the worm wheel of the machine by a shaft with universal couplings.

As first set up the instrument had the disadvantage of a very crowded scale at the low density end, and the deflections, though approximately proportional to the square of the transparency of the negative, needed close calibration if they were to be converted into transparencies. This peculiarity of the Koch instrument is in some cases, as for instance in measuring the absorptions of stellar images, an advantage; in this case it served merely to render more prominent the grain of the plate. A subsidiary lighting was therefore introduced, a new cell with two windows having been constructed for the purpose, and the anode of the low potential cell charged to about 6 volts, as shown in figs 1&2. The effect is to use a part of the voltage-current relation of this cell at a point where it is nearly linear, so giving a close approximation to an evenly divided scale of transparencies.

For some negatives the denser parts were still too closed in, hence a neutral-tinted glass transmitting about one-third of the light was introduced into the beam, mounted on a "gate" so that by the touch of a lever it could be removed or re-inserted. When the deflection exceeds about 70% of the scale the removal of the glass allows the densities above 0.5 to be recorded on a scale which gives easily measurable results up to densities of 1.5. The great sensitiveness of the Koch photometer, which allowed of this sacrifice of two-thirds of the light, is noteworthy - the period being lengthened to just over one second, that is to say, it remains shorter than that of the best thermo-electric instrument.

The chief error in Koch's original instrument was a rise in the deflection following prolonged illumination. The cause of this rise has been investigated since; but at the time the instrument was constructed the investigations had either not appeared or were not accessible. Studies made by the writer at the time showed that the rise was of the nature of a lowering of the sparking potential caused by and depending on both the magnitude of the current passed and its duration, and that an additional cause of trouble in the early cells was due to the large area of window surface. This led to the use of cells of small capacity to allow of smaller currents and to their use at lower voltages where the variation of the sparking potential is of less moment. The area of window surface was reduced to a few square millimetres. The success of these measures is shown in the following comparison with Koch's results:-

	Time from commencement of exposure (seconds):					
	0	*2*	*5*	*10*	*30*	*60*
Edinburgh instrument	3.8	98.0	98.0	98.1	98.2	98.3
Cf. Koch	69.2	-	19.3	17.8	16.3	15.5

A further error in Koch's instrument, though apparently similar, is not so easily eliminated. It takes the form of a movement of the zero for a short time after the illumination is cut off the high potantial cell, and must be due to a dark current in that cell. Fig 3 shows a record of the effect, in this case the total motion is 1.3 mm. in a deflection of 100 mm, whereas Koch found a movement of 0.9 mm in a deflection of 55 mm. This effect is practically absent from vacuum cells. The record of fig. 3 gives a good idea of the speed of the instrument, and shows also the existence of another defect, an irregular slow to and fro motion fo the thread when the cell is illuminated. This is due to battery variations, and is usually masked by the grain of the plate in records of stellar spectra. As the figure shows, it is not always present.

The position of equilibrium of the thread depends on the equality of the currents in the cells, and since in the cells used the current is very closely proportional to the illumination it might be expected that the compensation for variations in the illuminant would be almost perfect. The colour of the light, however, changes with the current through the lamp, and two cells having exactly the same colour sensitiveness are not easily made; in addition, the colour sensitiveness depends also on the potential on the cell. Fig. 4 shows the effect of a large change in the lamp current - from 1.45 amps. to 1.15 amps., which, though dividing the cell currents by four, changes the deflection by only two per cent. A similar result would arise from defective insulation, but the use of amber thread supports and a sealing-wax coat at the point where the central electrode of the cell emerges are sufficient to make any such effect completely negligible.

A simple arrangement causes the instrument to ring a warning bell at the completion of a record, and to cut off the motor current.

In addition to its use for photometry the instrument may also be used for position measures of hazy spectral lines. The measures may be made either on a record taken in the usual way, or, better, by moving the screw by hand, and reading its divided circle when the thread image passes certain points (each tenth division, say) of the scale. As the spectral line passes over the slit the thread moves along the scale, reaches a maximum at the centre of the line, and then re-traces its path. It is possible to read the position of the maximum with considerable accuracy, but in the case of symmetrical lines much greater precision is obtained by using the mean of two positions of equal density on either side of this maximum, where the thread may be moving perhaps two divisions for each 1μ moved by the negative. For this method of using the instrument it was found convenient (1) to reflect the image of the thread on to a scale placed immediately above the divided circle of the screw; (2) to have mounted on the carriage a negative similar to that to be measured, viewed by a microscope fixed to the base of the instrument, and with the lines to be measured marked on it; and (3) to use as high a speed of action of the thread as possible. A period of one second is too long for rapid work - the thread must follow practically at once the movement of the screw, and it is desirable to have a period not greater than one-fifth of a second. With these arrangements the readings may be made as fast as they can be copied down by an assistant.

Instances of work where the great accuracy of setting obtainable by the photometric method is of real value have as yet not presented themselves. There appeared to be a promising field in the case of plates taken in the third order of a 21½ ft. Rowland grating spectrograph for the purpose of determining the solar rotation by the Doppler method. In this case the comparison lines are atmospheric absorption lines in the same spectrum, so that the complications introduced by the usual comparison spectrum do not arise. In these spectra, which are taken by Mr. J. Storey in the course of his work on the subject,* the solar lines in particular are broad, and it is only by long practice that consistent settings on them can be made by eye to within 10μ. In the readings given below it should be noticed that since the slit is not more than 30μ broad each reading refers to a independent area of the negative.

Scale div.	*Typical solar line Approaching*	*Receding*	*Sum*	*Typical atmospheric line Approaching*	*Receding*	*Sum*
20	$0^r.300$	$0^r.907$	1.207	$0^r.640$	$0^r.972$	1.612
30	.368	.833	.201	.678	.937	.615
40	.413	.785	.198	.706	.911	.617
50	.458	.740	.198	.730	.887	.617
60	.503	.692	.195	.756	.861	.617

Since the above sums giving the positions of the lines have to be divided by a factor of about four to reduce them to millimetres it is evident that settings are easily made to within 1μ by this method. The error due to the departure of the plate from a true plane, coupled with the tilt necessary in the Littrow type of mounting, is however responsible for line shifts of the order of 5μ** so that actually the gain in accuracy is small. The only systematic difference traced between eye and photometer measures was in the case of the line 6310.101 attributed to Oxygen (atmospheric) which was shown by the photometer to be compound, having a weak solar line almost coinciding with it.

* Monthly Notices, R.A.S. 71, 1911, p. 674

** See *Eagle, Astrophys. Jour.* 31, 1910, p. 136.

Experience of position measures of this nature have led the writer to believe that it is undesirable to record spectra with an arrangement giving large linear magnification. The true field of the photometer is in translating the density differences of the negative into the more easily measured differences in the position of the thread, but the actual measurement is the province of the measuring machine. Mechanical methods of magnification are usually inferior to optical methods, which may be applied before the record is made, and there seems no object in recording a negative unless its features are too hazy to be measured by eye. Given a negative of this kind, the record (in the writer's opinion) should be made on a plate fixed to the same moving carriage as the negative. This at once removes the errors of the recording screw, and allows the full length of the negative to be recorded for subsequent measurement on an accurate machine, which will in all probability be available in any establishment requiring records of this nature. The expense of accurate mechanical parts in the photometer is avoided, and features may be compared which are well separated on the negative without the necessity for step-by-step methods of bridging the difference, for it is obvious that with a magnification of 20× not more than one inch of the negative can be included on a single record. The sole difficulty is in producing an image of the thread sufficiently well defined for the purpose, and this appears to be merely a matter of suitable optical parts.

Other improvements which may be suggested for position work are the use of vacuum (preferably Sodium) cells in place of gas-filled cells, and of a sensitive electrometer with one plate of low capacity, so that the thread may be charged to a high potential and this plate connected to the cells.

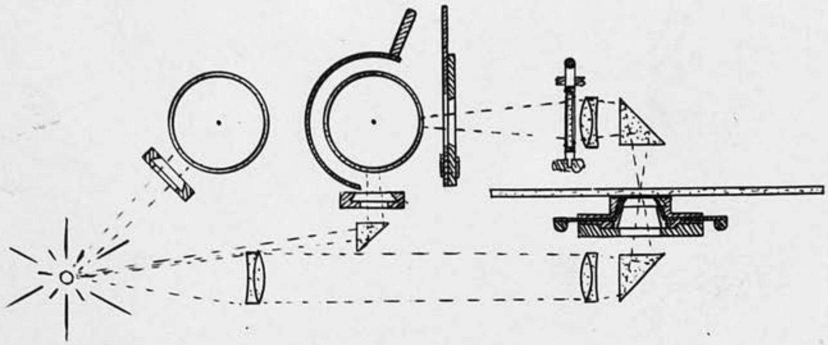

Fig. 1, Optical Arrangements.

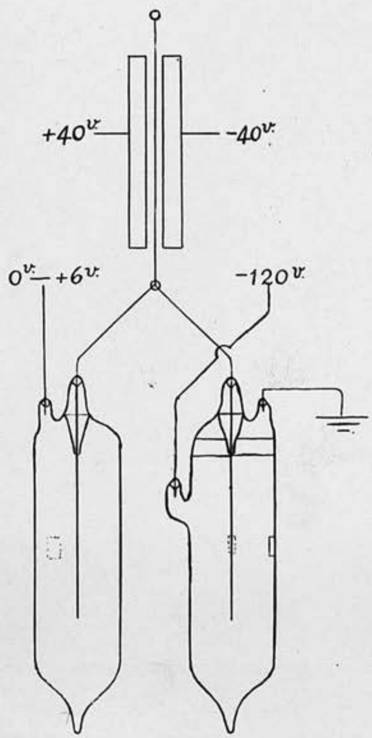

Fig. 2, Electrical Connections.

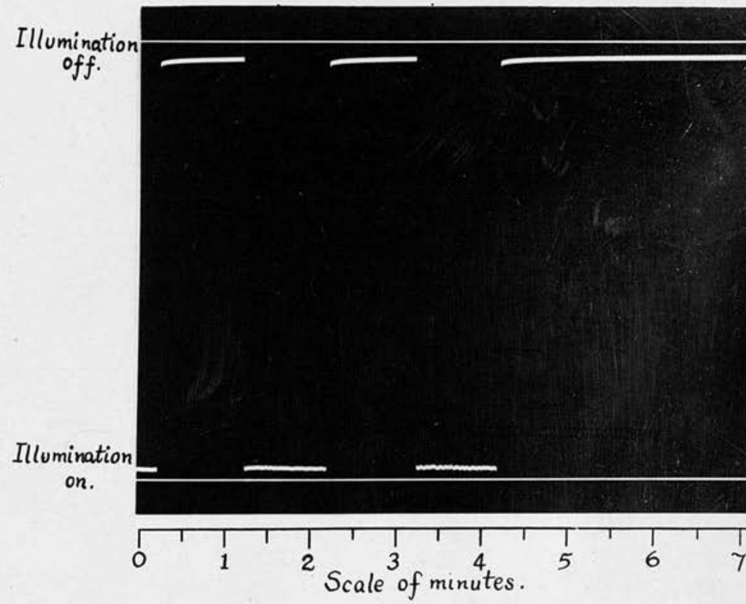

Fig 3. Showing effect of alternate illumination and obscuration.

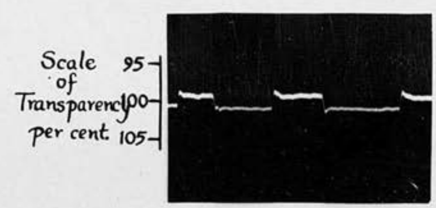

Fig. 4. Effect of changing current through lamp from 1·45 to 1·15 amps.

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SESSION 1924–1925.

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The Law of Blackening of the Photographic Plate at Low Densities.

By E. A. Baker, B.Sc.

(APPENDIX 1, PARTS I - III.)

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XV.—The Law of Blackening of the Photographic Plate at Low Densities. By E. A. Baker, B.Sc. *Communicated by* Professor R. A. Sampson, F.R.S.

(MS. received January 12, 1925. Read January 12, 1925.)

I. Plan of the Investigation.

The following paper is ancillary to certain investigations in stellar photometry which are in progress at the Royal Observatory, Edinburgh, and require the establishment of data relating to the blackening of the plates employed. It describes the laboratory work carried out for this purpose, generally over the very restricted range of exposure times and densities used in the stellar work, but occasionally extended to cover a wider field. The conditions are chosen in order to give the maximum accuracy and range to the stellar work and not with a view to highly accurate laboratory work; on the other hand, the periodical change of emulsion which must occur in any lengthy investigation introduces an element which, though of considerable interest, is lacking in most of the published work on the subject. The plates used are Ilford Special Rapid Panchromatic, Imperial "B" Panchromatic, and Wellington Spectrum Panchromatic, all rapid panchromatic plates differing little in speed and having a sharp fall in sensitiveness at 660 $\mu\mu$. The stellar negatives cover a range of density from 0·05 to 1·0 and of wave-length from 380 $\mu\mu$ to 660 $\mu\mu$; and though their exposures range from a minute to an hour or more, yet in consequence of the small width of the image which is trailed over the plate the actual exposure time of any particular region of the plate is only one-twentieth of the total exposure time.

I would refer first to the method employed by Professor Sampson in the above investigations, as it determines the notation and to some extent the treatment that follows. Assuming a uniform emulsion and constant working conditions, the four quantities concerned are the illumination I of the plate, its wave-length λ, the duration t of the exposure and the resulting density D. In a preliminary paper * Professor Sampson made certain assumptions as to the relation between these four quantities; but in his later work † he makes no such assumptions, using

* R. A. Sampson, *Monthly Notices R.A.S.*, **83**, 1923, p. 174.

† *Ibid.*, **85**, 1925, p. 212.

the perfectly general expansion of log I in terms of two quantities Δ and T, thus:—

$$\log_{10} I/I_0 = a\Delta + bT + \tfrac{1}{2}c\Delta^2 + d\Delta T + \tfrac{1}{2}eT^2 + \ldots \qquad (1)$$

where Δ is a function of D only, $T = \log_{10} t$, and I_0 and the coefficients $a, b, c \ldots$ are functions of the wave-length λ. As Professor Sampson points out, if Δ is replaced by D the first two terms of the expansion (1) give Hurter and Driffield's formula as modified by Schwarzschild. The form chosen for Δ in this paper is $\log_{10}(10^D - 1)$ or $\log_{10}(O - 1)$ where O is the opacity; this approaches D so rapidly that above unity Δ and D are practically interchangeable. As so defined, Δ is zero at about the middle of its range. A similar result is secured for T by choosing the unit of time as 30 seconds.

To arrive at definite values for the coefficients in (1), the quantity I_0 which is not required is eliminated by differentiation and two quantities p, q are introduced, defined by the equations

$$\left.\begin{aligned} p &= -\frac{\partial(\log I)}{\partial(\log t)} = -b - d\Delta - eT - \ldots \\ q &= +\frac{\partial(\log_{10} I)}{\partial\Delta} = a + c\Delta + dT + \ldots \end{aligned}\right\} \qquad (2)$$

It will be noticed that p is identical with Schwarzschild's index, while over a higher density range the gamma of the plate, taken as usual with reference to the exposure time, is given by $\frac{p}{q}$. By plotting p and q, first against Δ for $T = 0$ (*i.e.* for 30 seconds' exposure time) and then against T for $\Delta = 0$, curves are obtained which, in so far as they can be regarded as straight lines, give the coefficients in (1) as far as those of the second degree terms, and indicate by their departures from straight lines the magnitudes of the higher terms of the expansion. When this process had been carried out for several batches of plates it became clear that the coefficient d is small, and generally that the terms of (1), involving both Δ and T, are negligible in the stellar work; but that for some emulsions and wave-lengths higher powers of Δ than the second are needed to express the results with accuracy. Accordingly Professor Sampson works with a formula equivalent to

$$\log I/I_0 = \theta(\lambda, D) + \phi(\lambda, T) \qquad (3)$$

and showed that by his method of reduction the term involving T is eliminated to a remarkable extent, completely, in fact, if that term can be put in the form

$$\phi(\lambda, T) = A(\lambda) + B(T) + \lambda C(T).$$

This is equivalent to the condition that, for any value of T within the range used, the relation between p and λ is linear. It is then possible to put the numerical results as far as they are required for application to the stellar spectra in the form of curves connecting log I with the deflection of the Koch photometer by which the spectra are recorded, so eliminating all reference to Δ or D. In what follows, however, no presupposition is made as to the special value of d or any other coefficient.

The measurement of p and q thus involves varying the time, the illumination, and the value of $(O-1)$ in known ratios. An accuracy of 1 per cent. is aimed at, hence the measurement of time-ratios involves no difficulties, at any rate for times over ten seconds. In this work it is more important to have a continuous and well-defined scale of opacities than one which is theoretically correct, and the scale used is to some extent arbitrary though closely approximating to the theoretical scale of "opacities in parallel light." It is obtained by throwing a beam of light of fixed relative aperture ($f:6$) on to the plate and measuring, by a photo-electric cell, that portion of the light emerging within an equal relative aperture. Illumination ratios are measured by superposing two nearly equal illuminations, so giving an illumination double the mean of the separate illuminations. The practical difficulties involved in this fundamental method are very slight compared, for instance, with the elaborate screening arrangements required for the inverse square law method and the difficulty of realising a point source; while the illumination ratio 2 : 1 is specially appropriate to the present work, for it is sufficient to allow of accurate measures and yet so small that the variations of p and q over the corresponding differences of T and Δ are insensible.

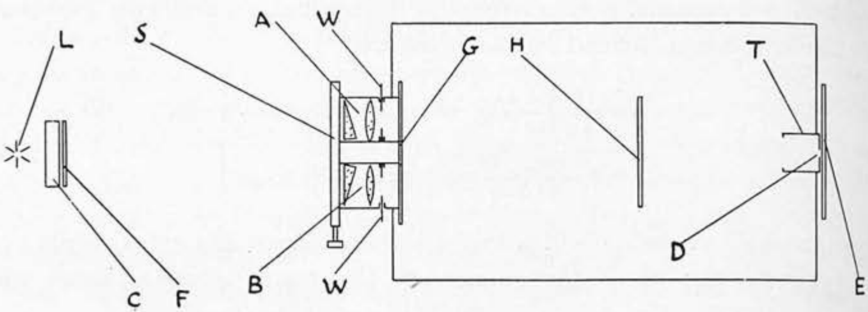

FIG. 1.—Section of camera.

The camera used is shown in fig. 1. It consists of a light-tight box about 20 inches long by 9 inches square, with two similar lens-prism systems A, B designed to throw two beams of light from the lamp L towards the plate E. A hand shutter S covers or exposes both lenses; but either beam may be occulted by substituting a piece of blackened brass for one of the Waterhouse stops W, W. Except for experiments involving high illuminations, the lamp is kept well out of focus; and to give uniform illumination over the diaphragm D, which limits the area of plate exposed, ground glass is inserted at G and H. The plate, of lantern size $3\frac{1}{4}$ inches square, is held in a double-slide plate-holder almost in contact with the aperture D, which is $\frac{1}{8}$ inch in diameter, so that by moving the plate by $\frac{1}{4}$ inch after each exposure in one direction or the other 100 exposures are made on each plate. Later, in place of the single tube T, four such tubes were inserted with apertures spaced at the corners of a square of $1\frac{1}{2}$ inches side, so that four regions of the plate, distinguished by making the apertures of different shapes, are exposed simultaneously, a full plate having 25 sets of exposures.

The source of light is a 100-cp. Pointolite lamp fed by a battery of large storage cells and controlled by a rheostat and a standard ammeter. Different regions of the spectrum are isolated by inserting at F one of the Wratten filters 71 to 76 (monochromatic series) or 18 (ultra-violet). In the later arrangement four filters can be used simultaneously by placing one in front of each of the four tubes T. A cell of N/5 copper sulphate solution 2 cm. thick is placed at C to make the effects of the filtered regions comparable: without this cell the red and orange filters give results greatly in excess of the blue and violet. So far as can be seen in a small direct-vision spectroscope this cell removes the whole of the deep red light otherwise transmitted by some of the green and blue filters. The wave-lengths used in Table VI are deduced on the assumption that the curves given by the makers to show the transmissions of the filters represent also the effective composition of the light passed by them with the copper sulphate cell in place, neglecting light at the red end beyond 670 $\mu\mu$. Stray light is eliminated by enclosing the lamp in a lantern (which must be well ventilated, for the lamp is unsteady when too hot), and by working in a dark room. The exposure times are taken from a deck watch with a luminous centre seconds hand, which moves with a sharp click at one-second intervals.

Values of q are obtained by exposing to lenses A, A+B, B in succession for equal times t, moving the plate after each exposure. Calling the corresponding illuminations I_1, I_1+I_2, I_2 and the resulting blackenings Δ_1, Δ_3, Δ_2, the unavoidable difference between I_1 and I_2 may be allowed for by assuming p and q constant over small intervals of T and Δ. In such cases equations (2) give on integration

$$\log I/I' = q\Delta - pT \qquad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (4)$$

Putting this equation in the equivalent form

$$c \cdot \exp\left(\frac{q\Delta}{\mu}\right) = \mathrm{I},$$

where μ is the modulus of common logarithms and $c = \mathrm{I}' \exp\left(-\frac{p\mathrm{T}}{\mu}\right)$, we have, if Δ' is the blackening which would result from an exposure to the illumination $\frac{1}{2}(\mathrm{I}_1 + \mathrm{I}_2)$ for time t,

$$\begin{aligned} c \exp\left(\frac{q\Delta'}{\mu}\right) &= \tfrac{1}{2}(\mathrm{I}_1 + \mathrm{I}_2) \\ &= \tfrac{1}{2}c\left\{\exp\left(\frac{q\Delta_1}{\mu}\right) + \exp\left(\frac{q\Delta_2}{\mu}\right)\right\} \\ &= c \exp\left(\frac{q(\Delta_1 + \Delta_2)}{2\mu}\right) \cosh \frac{q(\Delta_1 - \Delta_2)}{2\mu}, \end{aligned}$$

so that

$$\Delta' = \tfrac{1}{2}(\Delta_1 + \Delta_2) + \frac{1}{q} \log_{10} \cosh \frac{q(\Delta_1 - \Delta_2)}{2\mu} \quad . \quad . \quad . \quad (5)$$

The final term, which is given with sufficient accuracy by a very rough knowledge of the value of q, seldom exceeds 0·01. The value of q corresponding to the time t and to the mean blackening $\frac{1}{2}(\Delta_3 + \Delta')$ is then $\frac{\log_{10} 2}{\Delta_3 - \Delta'}$. Values for other densities and the same exposure time are found by repeating this process, either with another pair of Waterhouse stops, the areas of which increase roughly in the ratio 2 : 1 from 1 to 64, or for lower illuminations with a neutral absorbing screen (*e.g.* a fogged process plate developed with quinol) in front of the lamp. A 30-cp. Pointolite lamp was occasionally used.

The values of p are found by exposing successively to I_1 for time t, to $\mathrm{I}_1 + \mathrm{I}_2$ for time t_4, and to I_2 for time t, choosing the value of t_4 such that the blackenings Δ_1, Δ_4, Δ_2 obtained are approximately equal. Putting as before T for $\log_{10} t$ the value of p corresponding to the blackening Δ_4 and the mean of T and T_4 is given by

$$p(\mathrm{T} - \mathrm{T}_4) = \log_{10} 2 + q(\Delta' - \Delta_4)$$

where Δ' is given by (5). Here the value of q is supposed known with all the accuracy required from experiments on plates of the same batch if not on the same plate.

The blackening of the deposits is measured by an automatic densitometer designed in the first place for measuring the sizes of stellar images. Two photo-electric cells A, B (figs. 2–4) are connected together, and to one plate of an electrometer C, as in Koch's microphotometer.* The ball of

* P. P. Koch, *Ann. d. Physik*, 39, 1912, p. 705.

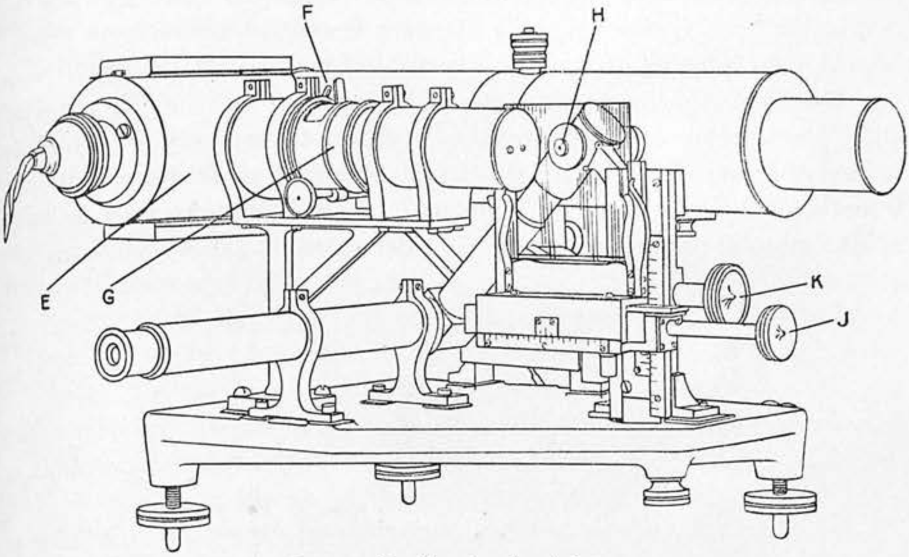

FIG. 2.—Densitometer, front view.

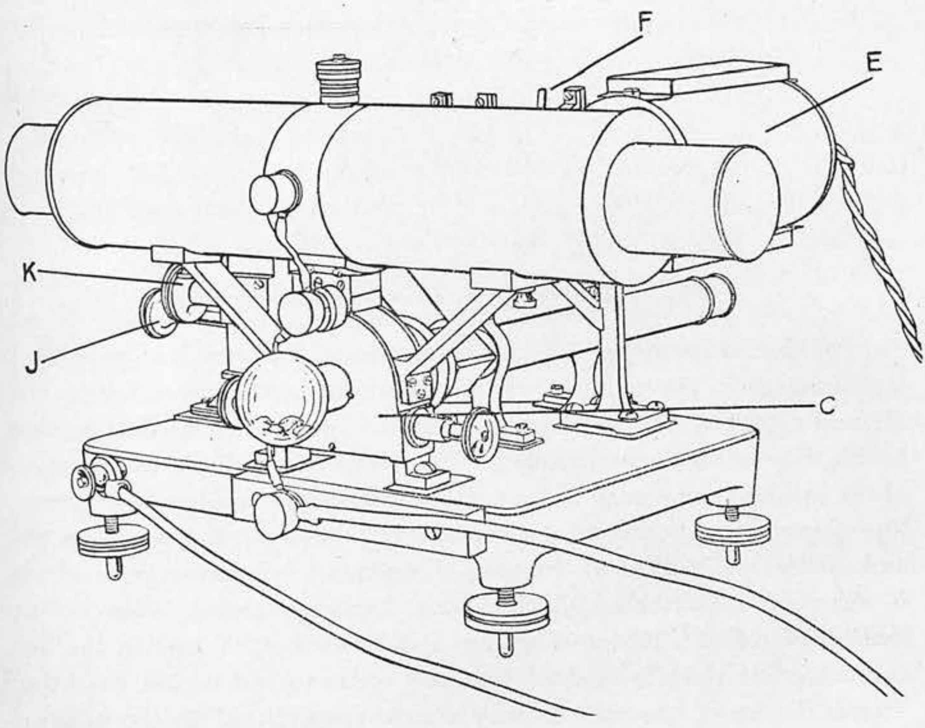

FIG. 3.—Densitometer from rear, showing electrometer.

a Pointolite lamp contained in the ventilated lantern E illuminates cell A directly through the adjustable aperture F, and is focussed by a Petzval lens G on a pinhole H immediately behind and almost in contact with the film of the photographic plate. The last-named is held in a double-slide plate-holder and can be moved either horizontally or vertically in its own plane by the milled heads J, K. Part of the emergent light is reflected from the face of a prism L of small angle to give an image of the pinhole magnified about 50 diameters in the eyepiece of the

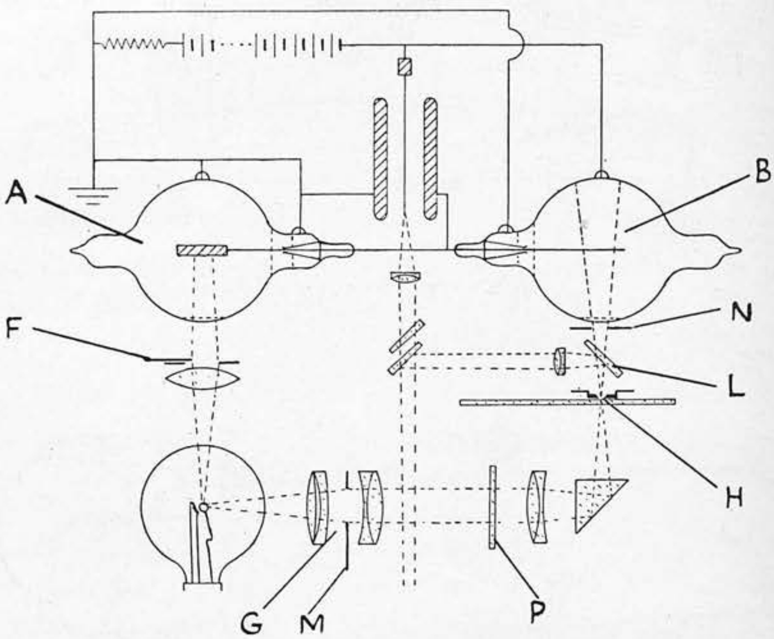

FIG. 4.—Densitometer, electrical and optical arrangements.

electrometer microscope. This arrangement, intended for centring a stellar image in the pinhole, was found useful in the present work as it allowed a part of the film free from slight defects such as dust particles to be selected for measurement The stops M, N limit the apertures of the incident and emergent beams to the ratio f : 6, as already mentioned. The electrometer is described elsewhere *; in this work it has been used as a deflection instrument, keeping the plates fixed and using a battery of fifty large Leclanche cells to charge both the moving wire and the cathode of cell B. The readings are independent of changes in the lamp current, while the wire is dead beat and comes to rest within one second. The deflection of the wire is very nearly proportional to the square of

* *Journ. Sci. Instruments*, 1, 1924, p. 345. It was made in the Observatory.

the transparency, giving a very open scale at the low density end. Higher densities than 0·5 are obtained by substituting for the neutral glass P (density 0·95) either of two similar but more transparent glasses (densities 0·39, 0·06).

The scale was calibrated by taking advantage of the fact that the cell B is known to give currents very closely proportional to the illumination. It was disconnected from cell A and connected to a Broca galvanometer with a standard universal shunt, when by using in place of the normal pinhole of 1 mm. diameter an aperture of about 5 sq. mm. area the deflections of the galvanometer were large enough for measurement with an accuracy of 0·5 per cent. The variations of the lamp were eliminated by taking a sufficiently large number of measures. In this way a series of fogged strips were measured and were subsequently used as standards for calibrating both this instrument and the Koch photometer used for recording the stellar spectra. No changes in the scale can be traced from the start of this work. The cells are of the sensitised helium-filled type and were made in 1918; no record has been kept of their absolute sensitiveness, but changes in the calibration of the scale have nowhere exceeded 2 per cent. since the first calibration some six months after the cells were made, and there has been no apparent decrease in the speed of working such as would have followed any considerable fall in sensitiveness.

The thread image is brought to the zero (opacity) end of the scale, first when the thread and plates are earthed, by moving the microscope, and again after charging the thread and cell, by moving the plates, with the result that the zero is independent of variations in the potential of the thread. The aperture F is adjusted to give the full deflection ($\Delta = -\infty$) for the clear plate in the immediate neighbourhood (*i.e.* within 2 mm.) of the deposit to be measured, and the deflection for the deposit is read; this is repeated, using as standard the clear plate on the opposite side of the deposit, and the mean of the deflections is noted. This method of dealing with variations of density over the clear parts of the plate is not ideal; it does not eliminate the effect of fog which is measurably greater towards the edges of even freshly obtained plates, especially those packed with paper separators, and it is unsuited to the measurement of high densities where the Eberhard development effect * first becomes prominent and for still higher densities is masked by fog due to scattered light (halation). For very low densities, however, where the variation in density of the clear plate over a few centimetres may be

* G. Eberhard, *Phys. Zeit.*, 13, 1912, p. 288.

much greater than the effect of the exposure, it is necessary to adopt some such method. Readings of Δ are taken to 0·01 between $\bar{2}$ and $\bar{1}$ and to 0·002 from $\bar{1}$ upwards.

The plates are developed in open dishes rocked by hand, first bringing dish and developer to 65° F. The time is fixed separately for each batch of plates, the aim being the maximum density consistent with freedom from fog. The development formula given by the makers of the plates has almost invariably been used—for the Ilford plates, pyro-soda; and for the Wellington and Imperial plates, metol-quinol. The makers' tables have been found useful guides in finding the most suitable times of development; more than this is not to be expected from these tables since different lots of developer chemicals, even though obtained from the same source, cannot be relied upon to give the same results.*

Many of the plates have been well washed in running tap water and dried before exposure. This procedure, recommended by Messrs Ilford for increasing the red and green sensitiveness,† produces very different results on different emulsions, especially in regard to their keeping properties, for while some batches of plates have kept for over a month after washing, retaining all the while a greatly increased sensitiveness, others have given little increase in sensitiveness or have fogged within a few days. As a rule the increased sensitiveness has been found to be greater than the 20 to 30 per cent. stated by Messrs Ilford. One batch of Imperial plates for which exact measures were made had their sensitiveness increased between 80 and 90 per cent. in the red and yellow, the sensitiveness in the blue and ultra-violet remaining unaffected. Tap water appears to be more effective than distilled water, possibly owing to its greater alkalinity (P_H 7·5); but the more strongly alkaline solutions tried—lime water of P_H 9·5 and N/50 sodium carbonate solution—led to fog and appeared to give no further increase of red sensitiveness, while a borax-boracic acid buffer solution (5 grms. boric acid, 2 grms. borax per litre) of P_H 8 gave the same relative increase of red sensitiveness but a slight all-over decrease of sensitiveness for the same development time consequent on the known restraining properties of borax on the developer.‡ These experiments were made by soaking the plates with frequent rocking for 15 minutes, for it has been found that little advantage, if any, is obtained by prolonging the washing or by the use of running water.

The accuracy of the measures depends almost wholly on the evenness

* *Cf.* L. A. Jones and E. Huse, *Opt. Soc. Amer. Journ.*, 7, 1923, p. 1086.

† *Panchromatism* (3rd edition), 1922, p. 18.

‡ *Brit. Journ. Phot.*, 1921, p. 639.

of the clear plate in the neighbourhood of the image, and it is seldom that the measures of Δ can be relied upon to 0·01 even for values above $\bar{1}$. Occasionally, however, plates are obtained which are exceptionally uniform over the unexposed parts and which give consistent results within the accuracy of 1 per cent. expected from the lamp and timing arrangement. The method of obtaining the values of p and q by a comparison of an image with its two neighbours goes far to eliminate irregularities of development, etc., such as have been investigated by Bloch * and others. None of the washed plates has given exceptionally uniform readings.

Although two plates from the same batch and subjected to the same treatment give consistent values for q, the value of p appears to depend to a considerable extent on the age of the plate or on some other factor which varies from month to month. As an example, the values of p for plates 44 and 45 given in Table VIII may be compared with those for the same batch and other circumstances given in Table VI. The former plates were taken in August 1924, the latter in April of the same year; and in the interval the value of p appears to have fallen by 0·06.† For this reason the values of p and q tabulated below are as far as possible from single plates; indeed it appears that the thirty or forty values of p or q given by one plate afford more reliable data on a particular point than a mass of less homogeneous material.

II. Results for Panchromatic Plates and Filtered Light.

The effects of varying the development conditions will first be considered, and to show clearly the contrast in this respect between high- and low-density work some figures given by Sheppard and Mees ‡ for a Barnet Photomechanical plate developed for 4 and for 8 minutes are shown in figs. 5 and 6, using first the given values of D, and then the corresponding values of Δ as ordinates. In spite of the close connection between γ the development factor and $\frac{1}{q}$ it is evident that for densities below 1·5 q is almost independent of the duration of development. Results obtained here from two Imperial plates, equally exposed but developed, one for $1\frac{1}{2}$ minutes, the other for 4 minutes in pyro-soda, are plotted in fig. 7, separately for the

* O. Bloch, *Phot. Journ.*, 1921, p. 425; H. T. Stetson and E. F. Carpenter, *Astrophys. Journ.*, **58**, 1923, p. 36; Mlle Clavier, *Sci. et Ind. Phot.*, **4** M, 1924, p. 9.

† Messrs Wellington and Ward, who were consulted on this point, state that they expect their panchromatic emulsions to give slightly lower values of p after, say, six months.

‡ Sheppard and Mees, *Theory of the Photographic Process*, London, 1907, p. 282.

red (filter 71) and the ultra-violet (filter 18). The main effect of increasing the development is to diminish I_0 of formula (1) in a ratio which

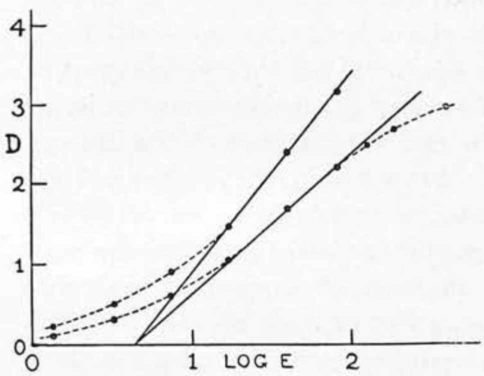

Fig. 5.—Barnet Photomechanical plate, developed for 4 and for 8 minutes (Sheppard and Mees).

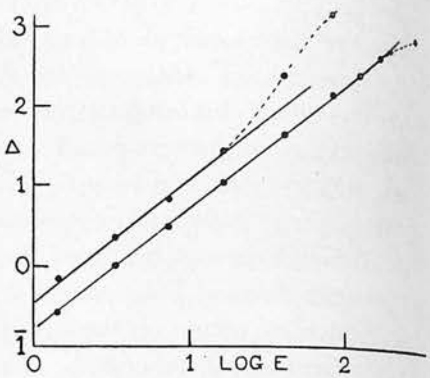

Fig. 6.—Result of replacing D of fig. 5 by Δ.

depends on the wave-length; q is slightly increased at low densities and diminished at high densities—that is to say, the coefficient c is affected by

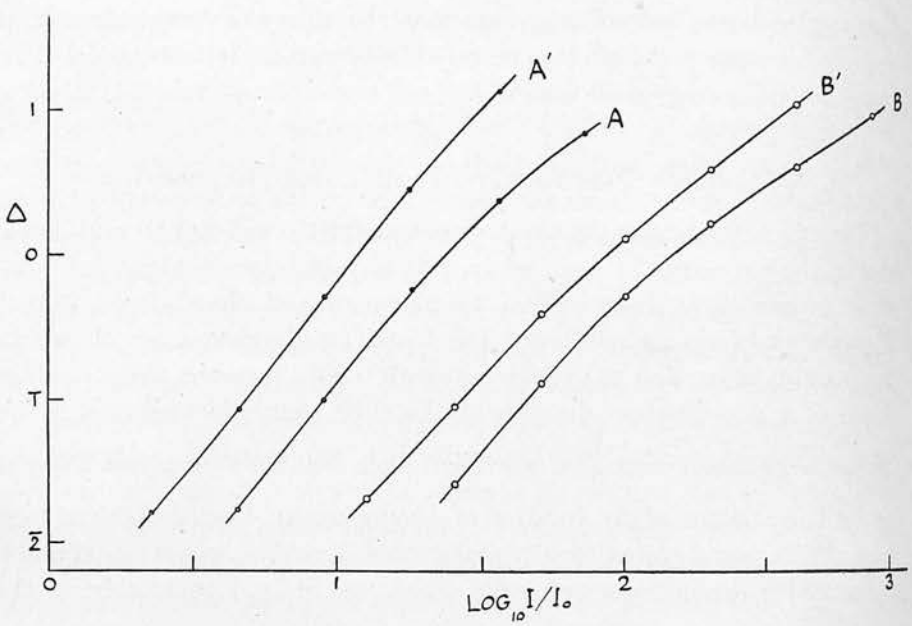

Fig. 7.—Result of increasing development time from $1\frac{1}{2}$ minutes (A, B) to 4 minutes (A′, B′).
A, A′, red light (filter 71). B, B′, ultra-violet light (filter 18).

development though the coefficient a is not. Temperature variations give exactly similar results. The result of omitting the potassium bromide from the developer was to confirm Sheppard and Mees's conclusion * that the effect of bromide in the developer is to move the characteristic curve as a whole parallel to the exposure axis; that is, to change I_0 only. The substitution of metol-quinol for pyro-soda left q entirely unaffected, both on Imperial and Wellington plates.

The effect of development variations on the values of p is easily measurable. An example is given in Table I of the effect of under-development.

TABLE I.

Plates 52, 53. Effect of development time on p or b.
Wellington 8896, pyro-soda developer.

Filter No.	71	73	75	18
Developed $1\frac{1}{2}$ min.	0·98	0·97	0·98	0·96
Developed 4 min.	0·95	0·93	0·93	0·93

Additional bromide in the developer produces a similar effect to under-development.

TABLE II.

Plates 59, 60. Effects of bromide in developer on p or b.
Imperial 4154, metol-quinol developer.

Filter No.	71	73	75	18
Imperial M.Q. with 3 grms. KBr per litre .	0·92	0·94	0·93	0·91
Same developer without bromide . .	0·91	0·91	0·88	0·86

Washing and drying the plates before exposure, in so far as no fog occurs, leaves the value of q unaffected by more than 2 per cent. Fog, whatever its cause, appears to lower the value of q.

The values of p are distinctly lower on washed plates.

TABLE III.

Plates 66, 67, 70, 71. Effect of previous washing on p or b.
Imperial 4585, metol-quinol developer.

Filter No.	71	73	75	18
Washed in tap water	0·88	0·85	0·80	0·85
Not washed	0·89	0·91	0·84	0·85

* *Loc. cit.*, pp. 112, 287.

Here, as in general, the values for the washed plates are somewhat erratic, and it is not certain that the effect on p resembles the increase of sensitiveness in being confined to the red end of the spectrum.

The determination of the variations of q with wave-length is on a less satisfactory basis than the remainder of the work. The convenience of working with the quantities p and q lies in their comparatively slow variations with the density and time, but in the variation of q with wave-length it is fairly certain that round about H_β ($486\,\mu\mu$) a large change occurs over a wave-length-interval of the same order as that involved in changing from one filter to the next. In the orange about $600\,\mu\mu$ (compare, for instance, the values for q in Table IV, filters 73 and 71) and in the violet about $400\,\mu\mu$ q changes only very slowly if at all with wave-length, and its values are definitely measurable; but in the green about $500\,\mu\mu$ the effective wave-length of the filtered light becomes indefinite, depending not only on the sensitiveness of the particular batch of plates over this region but also, by reason of the variation of q, on the density of the deposit. Experiments are in progress on this point with monochromatic light, but in the meantime it is only possible to give results with filtered light in the orange where q appears to have a minimum value, and in the violet and ultra-violet where q rises slowly (corresponding to lower contrast) as the wave-length decreases. There is apparently little change in q at $660\,\mu\mu$ where the sensitiveness of the plates falls sharply with increase of wave-length; but here again filtered light is unsatisfactory.

The manner in which q varies with the value of Δ for constant exposure time can be deduced from curves such as are shown in figs. 5-7; but it appears more clearly when q is plotted against the corresponding mean value of Δ, as in figs. 8 and 9. The forms of the curves appear to be characteristic of the makes of plate though their actual positions vary with the batch. The comparative flatness of the curves for the Wellington plates in fig. 9 is noticeable: it is of course possible to choose Δ such that any one of the curves in figs. 8 and 9 becomes a straight line parallel to the axis of Δ, but it appears on further examination (though since q depends on the choice of Δ this is not immediately obvious from the figures) that on the whole no marked flattening of the curves would result from another choice of variable to represent the blackening. The different values for q shown in fig. 9 for two batches of Wellington plates are by no means exceptional or confined to this make, and may be compared with the different values of p for different emulsions given in Table VI. It appears that q does not fluctuate more than about 30 per cent. from its mean value 0·5.

The question as to the dependence of q on the value of T at constant density is of importance in fixing the value of d. Since q depends to a

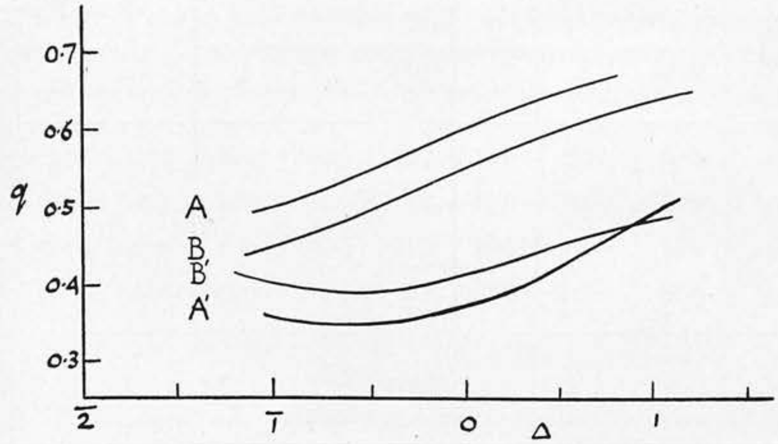

Fig. 8.—q as a function of Δ for constant exposure time 30 seconds.

A, Ilford plate, filter 18 (ultra-violet); A′, Ilford plate, filter 72 (orange); B, Imperial plate, filter 18 (ultra-violet); B′, Imperial plate, filter 72 (orange).

considerable extent on the value of Δ, and since particular values of Δ can be found only by a succession of trials on plates of the same batch, direct

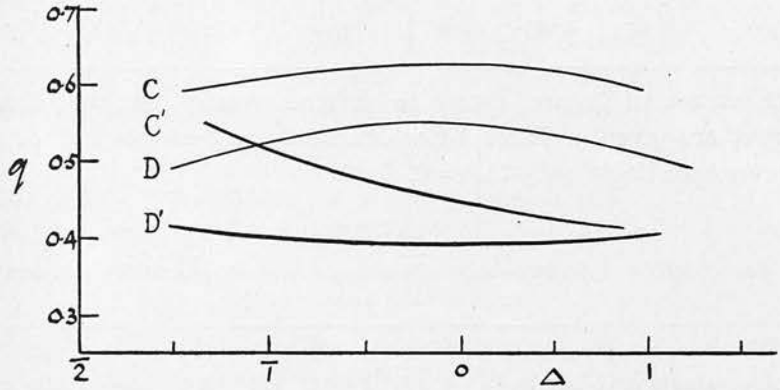

Fig. 9.—q as a function of Δ for constant exposure time 30 seconds.

C, D, filter 18 (ultra-violet); C′, D′, filter 72 (orange). Wellington plates.

information on this point is not easy to obtain. Tables IV and V give some results in cases where the densities for high and low values of T correspond almost perfectly. For comparison the corresponding values of p, deduced from observations on the same plates, are also given.

TABLE IV.

Imperial B 4585.

Time (seconds).	Filter No. 18.			Filter No. 73.			Filter No. 71.		
	Mean Δ.	q.	p.	Mean Δ.	q.	p.	Mean Δ.	q.	p.
20	0·43	0·627	0·85	$\bar{1}$·75	0·368	0·91	$\bar{1}$·97	0·368	0·90
45	0·43	0·640	0·83	$\bar{1}$·81	0·384	0·92	0·02	0·380	0·89
90	0·38	0·618	0·82	$\bar{1}$·79	0·380	0·91	$\bar{1}$·99	0·379	0·88
200	0·28	0·624	0·79	$\bar{1}$·62	0·384	0·88	$\bar{1}$·83	0·382	0·85

TABLE V.

Wellington 8896.

Time (seconds).	Filter No. 18.			Time (seconds).	Filter Nos. 71, 73 (mean).		
	Mean Δ.	q.	p.		Mean Δ.	q.	p.
25	0·00	0·536	0·96	11	$\bar{1}$·95	0·431	0·96
120	0·00	0·549	0·90	45	0·01	0·414	0·95
360	0·27	0·526	0·84	180	$\bar{1}$·90	0·408	0·91

The values of p are found to depend greatly on the emulsion. Examples are given in Table VI, which refers to densities and exposure times corresponding closely to $\Delta = 0$, $T = 0$.

TABLE VI.

p as a function of the wave-length for constant blackening $\Delta = 0$ and constant exposure time 30 seconds.

Filter No.	18	76	75	74	73	72	71
Approximate wave-lengths, $\mu\mu$	360	430	490	540	580	610	640
Ilford 5811 E	0·89	0·90	0·89	0·90	0·91	0·92	0·92
„ 5869 E	0·78	0·78	0·77	0·81	0·82	0·82	0·84
Imperial 4154	0·90	...	0·93	...	0·93	...	0·92
„ 4192	0·83	0·84	0·82	0·85	0·87	0·86	0·85
„ 4492	0·86	0·88	0·92	0·92	0·92	0·92	...
„ 4585	0·83	...	0·83	...	0·88	...	0·86
Wellington 8896	0·92	...	0·93	...	0·94	...	0·93

The extent of the variation with wave-length, which is small, appears to depend on the magnitude of p, being greater the lower the value of p. This is again evident in figs. 10 to 13. Figs. 10 and 11 show the relation between p and the exposure time for two regions of wave-length and for two batches of plates. It appears that while the coefficient b in (1) depends on the emulsion and varies little with wave-length, the

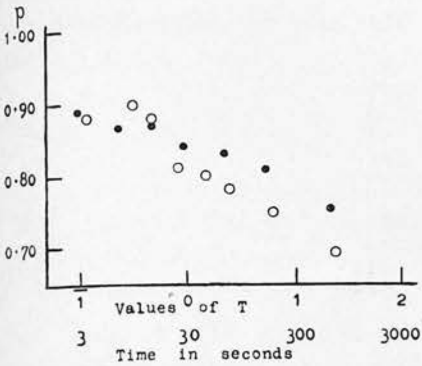

Fig. 10.—Imperial B 4192.

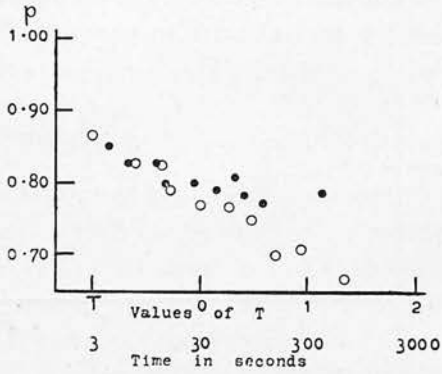

Fig. 11.—Ilford 5891 E.

p for constant density range, $\bar{1}{\cdot}5 - 0{\cdot}5$. Dots, filter 72 (orange); circles, filter 18 (ultra-violet).

coefficient c depends greatly on the wave-length and appears to have much the same values for different emulsions. Thus the results given in Table VI do not conflict with those of Hnatek * for Agfa Panchromatic plates and filtered light, for though he shows a considerable rise of p in the green and yellow his exposure times (2–5 minutes) are greater. A plate taken to test this point gave the following results:—

Table VII.

Values of p for various times and wave-lengths.
Wellington 8896.

Time (seconds).	Filter No.			
	18. ($\Delta=0{\cdot}0$).	75. ($\Delta=\bar{1}{\cdot}9$).	73. ($\Delta=\bar{1}{\cdot}6$).	71. ($\Delta=\bar{1}{\cdot}6$).
$7\frac{1}{2}$	0·980	0·972	0·972	0·952
30	0·945	0·949	0·958	0·944
120	0·887	0·935	0·941	0·923
450	0·824	0·879	0·878	0·832

* A. Hnatek, *Zeits. wiss. Phot.*, **22**, 1923, p. 177.

It appears that the increase of p in the green is appreciable only for longer exposures than occur in the stellar work.

The variation of p with the density for constant-time range is shown in figs. 12 and 13 for the same plate-batches and wave-lengths as in figs. 10 and 11. The coefficient d in (1) is evidently very small; but higher order coefficients become appreciable for densities greater than unity.

The constancy of p for constant exposure time and of q for constant density lead at once to equation (3). This may be compared with the

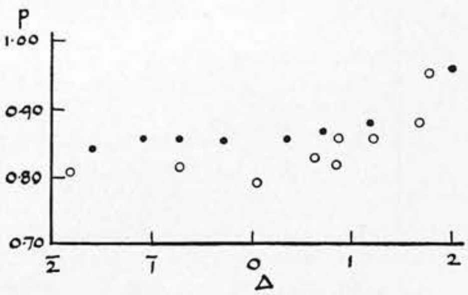

Fig. 12.—Imperial B 4192.

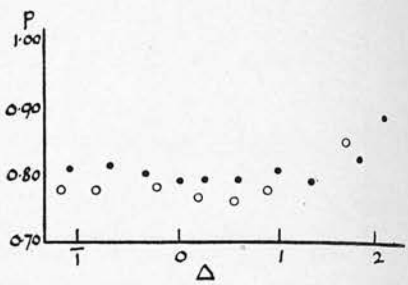

Fig. 13.—Ilford 5891 E.

p for constant time range 15–60 seconds. Dots, filter 72 (orange); circles, filter 18 (ultra-violet).

equation deduced by Dr J. Halm * from his own results and from those of Kron,† viz.

$$\log t = \phi(\mathrm{D}) + \psi(\log \mathrm{I}) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which asserts the constancy of p for constant illumination and the constancy of p/q for constant density. On the second point Tables IV–V give some information. Two plates taken, one with constant illumination, the other with constant time of exposure, gave the following results:—

Table VIII.

Imperial B 4192. Filter No. 18.

Illumination constant.				Time constant.		
Time (seconds).	T. (mean).	Δ.	p.	T. (mean).	Δ.	p.
$2\frac{1}{2}$–6	$\bar{1}$·15	$\bar{1}$·33	0·832	$\bar{1}$·82	$\bar{1}$·15	0·788
5–12	$\bar{1}$·40	$\bar{1}$·83	0·778	„	$\bar{1}$·83	0·776
13–30	$\bar{1}$·82	0·33	0·772	„	0·26	0·783
50–120	0·38	0·96	0·762	„	1·20	0·831
100–240	1·18	1·44	0·736	„	1·84	0·842

* J. Halm, *Monthly Notices R.A.S.*, **75**, 1915, p. 159.

† E. Kron, *Publ. des astrophys. Observ. zu Potsdam*, **22**, 1913, p. 67.

Even if the shortest exposures are ruled out as subject to systematic errors of timing, there is a definite fall of p with increase of exposure time. On the other hand, the observations at higher densities, in particular the rise of p with density shown also in figs. 12 and 13, are in accordance with (6).

It is concluded that the blackening of the panchromatic plates used in this work for exposures between 5 seconds and 10 minutes, wave-lengths between 360 $\mu\mu$ and 650 $\mu\mu$, and densities below unity, can be represented by the formula

$$\log I/I_0 = \theta(D) + bT + \tfrac{1}{2}eT^2$$

where T is $\log_{10}\left(\frac{\text{exposure time}}{\text{30 seconds}}\right)$. The quantity I_0 is known to depend on the emulsion, wave-length and development, but is left undetermined. The function θ depends on the emulsion and wave-length, but not on the development within wide limits. The coefficient b depends on the emulsion and to some extent on the development, but is almost independent of the wave-length; while the coefficient e depends on the wave-length but not to any great extent on the emulsion. A simple expression giving a good approximation to $\theta(D)$ even for densities below 0·01 is $\frac{1}{2}\log_{10}(O-1)$ where O is the " opacity in parallel light," so that for constant time

$$O = 1 + kI^2,$$

the constant k depending on the unit chosen for I and on the emulsion and development.

III. Bibliography of Work on the Validity of the Reciprocity Law.

The appended bibliography is devoted to work on the validity of the reciprocity law for photographic plates, and for references to the closely allied subject of the intermittency effect the second paper by A. Odencrants should be consulted. I am indebted to that paper for a certain number of the items, in particular for the reference to the paper by Fizeau and Foucault, who found the reciprocity law invalid when working with daguerreotypes. The work by C. V. L. Charlier is noteworthy since in it he uses the law of blackening published some ten years later by Schwarzschild and known by his name. Several papers which discuss the results of other workers without introducing fresh material have been omitted from the list.

As might be expected from the variety of emulsions used, the different workers arrive at conclusions which are apparently contradictory.

Several have tried, and failed, to find a connection between the value of Schwarzschild's index and the speed of the plate, and have concluded from their results that the index had a value independent of the emulsion. In the present work no very exact speed determinations have been made, but there is no indication of a connection between the value of p and the plate speed. On the other hand, several workers mention a variation of p with the batch of plates used. Many find that p is independent of the range of exposure-time used, but where such a variation is found it is always in the direction found here—a decrease as the times increase. It appears that those who have examined the point agree that p is higher for under-developed than for fully developed plates.

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(*Issued separately May* 14, 1925.)

1.—The Law of Blackening of the Photographic Plate at Low Densities. (Second Paper). By E. A. Baker, B.Sc. *Communicated by* Professor R. A. Sampson, F.R.S.

(MS. received November 10, 1926. Read December 6, 1926.)

IV. Results for Isochromatic and Blue-sensitive Plates and Filtered Light.

In a previous paper * the relation between the illumination I on a panchromatic plate, the duration t of the exposure, the wave-length λ of the light, and the resulting density D of the deposit on the developed plate was investigated. Starting with the expansion of log I in terms of functions of D, t, and λ in the form

$$\log \mathrm{I}/\mathrm{I}_0 = a\Delta + b\log_{10} t + \tfrac{1}{2}c\Delta^2 + d\Delta\log_{10} t + \tfrac{1}{2}e(\log_{10} t)^2 + \ldots \qquad (1)$$

where $\Delta \equiv \log_{10}(10^{\mathrm{D}} - 1)$, and a, b, c, d, e, etc., are functions of the wave-length λ, it was found convenient to express the results in terms of the two partial derivatives

$$p \equiv -\frac{\partial(\log \mathrm{I})}{\partial(\log t)} = -b - d\Delta - e\log_{10} t \ldots$$

$$q \equiv \frac{\partial(\log_{10} \mathrm{I})}{\partial\Delta} = a + c\Delta + d\log_{10} t + \ldots$$

Similar results are given below for isochromatic and blue-sensitive emulsions, using the same methods of working as are described in detail in the earlier paper. As before, filtered light has been used, and development has been prolonged up to the point at which further development would produce visible fog. The developing formula used is the pyro-soda formula recommended by the maker of the plates, and differs, therefore, according to the maker of the plate, and in some cases for different emulsions by the same maker. The instrument used for density measurement was described at length in the first paper; it utilises only a comparatively small central portion of the emergent light from the plate, hence the densities are not the usual "diffuse" densities corresponding to the value of the negative in contact printing, but approximately "specular" densities.

In the previous paper it was found that neither p nor q is greatly affected by variations in the development, provided that the density is low.

* These *Proceedings*, 45, 1925, p. 166.

The results now obtained correspond exactly with those for panchromatic plates, and the conclusions there drawn may be repeated with slight amplification:—The main effect of increasing the development is to diminish I_0 of formula (1) in a ratio which depends on the wave-length; q is slightly increased at low densities and is diminished at high densities—that is to say, the coefficient c is affected by development though the coefficient a is not; while p is diminished whatever the density at which it is determined, the coefficient b being affected, but not the coefficient d. These results are

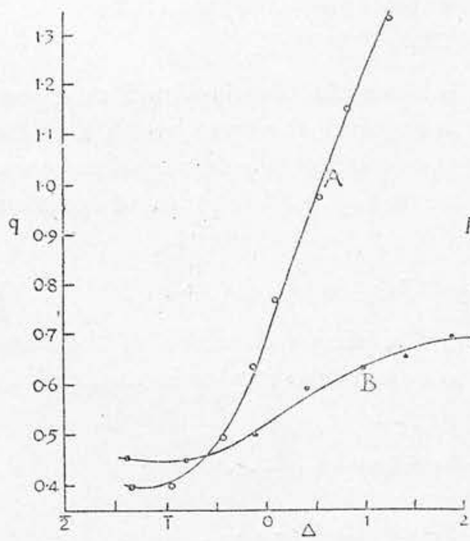

FIG. 1.—Relation between q and Δ for exposure time 30 seconds. Blue light.
A. Developed $2\frac{1}{2}$ min.
B. ,, 9 ,,

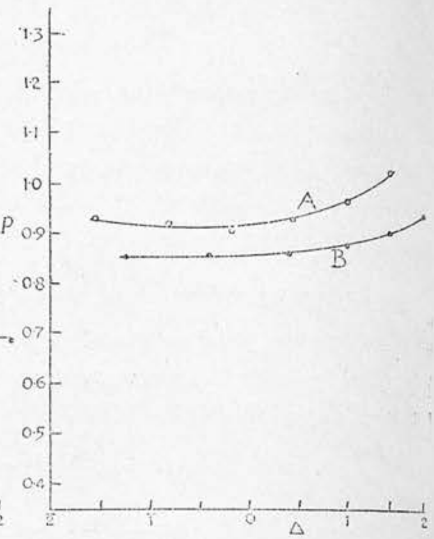

FIG. 2.—Relation between p and Δ for exposure time 30 seconds. Blue light.
A. Developed $2\frac{1}{2}$ min.
B. ,, 9 ,,

shown in the case of an Ilford "Zenith" plate (blue sensitive, speed 650 H. & D.) in figs. 1 and 2, for blue light and development of $2\frac{1}{2}$ and 9 minutes.

Typical curves showing the relation between Δ and log I are shown in figs. 3–6. From these the following conclusions regarding q may be drawn:—

1. At the lowest measurable densities (of the order 0·002) q is in the neighbourhood of 0·5, and does not depend greatly on the wave-length of the light.

2. As the density increases q becomes affected by the wave-length; the effect of wave-length reaches a maximum for densities between 0·2 and 0·4, but is still prominent at the highest densities examined.

3. The value of q is smallest and most constant when the light is yellow or green and the emulsion isochromatic. It is greatest for the ultra-violet, for which it increases rapidly with the density.

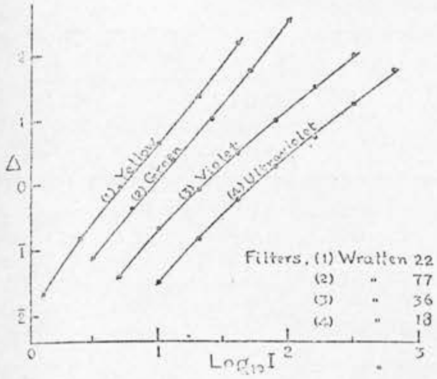

Fig. 3.—Relation between Δ and log I for Ilford Auto-filter plates. Exposure time 30 seconds.

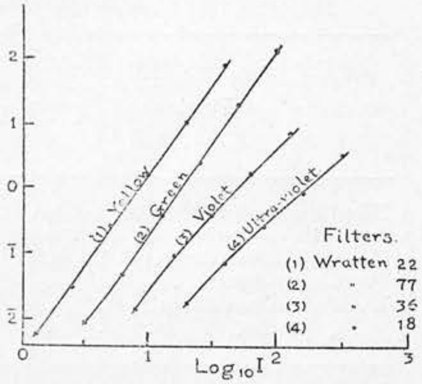

Fig. 4.—Relation between Δ and log I for Gevaert Filtered Ortho. plates. Exposure time 30 seconds.

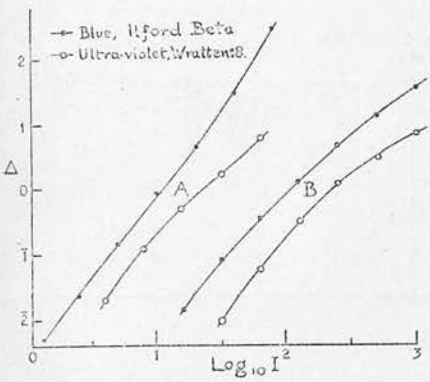

Fig. 5.—Relation between Δ and log I for
A. Ilford Process plates.
B. Ilford Empress plates.
Exposure time 30 seconds.

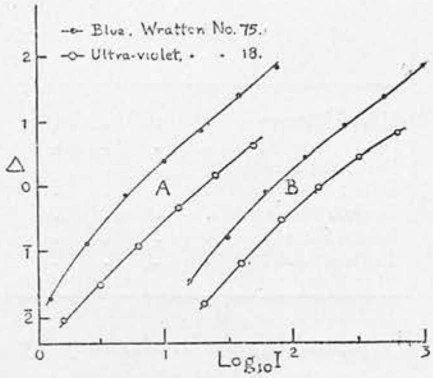

Fig. 6.—Relation between Δ and log I for
A. Ilford Zenith plates.
B. Imperial Eclipse plates.
Exposure time 30 seconds.

Note.—The zero of the log I scale in these figures is arbitrary. No comparisons between the relative speeds of the emulsions have been made.

The variation of q with the wave-length is shown in Table I.

The complete discussion of the quantity q should include its variation with log t; but since from the definitions of p and q we have

$$\frac{\partial q}{\partial \log_{10} t} = -\frac{\partial p}{\partial \Delta} = d + \text{third-order terms}$$

(*cf.* (1)), this variation may be deduced from the manner in which p varies with the density when the exposure time is constant. There is a

TABLE I.

Values of p and q for density 0·3 and exposure time 30 seconds.

1. *Isochromatic and Screened Emulsions.*

Plate.	Speed H. & D.	p (mean).	Values of q. Orange and Green.	Violet.	Ultra-Violet.
Ilford Screened Chromatic	270	0·90	0·38	0·54	0·58
Imperial S.S. Ortho.	300	0·91	0·38	0·42	0·54
Gevaert Filtered Ortho.	300	0·92	0·38	0·48	0·55
Ilford Auto-filter	400	0·95	0·43	0·55	0·60
Wellington Anti-screen	450	0·78	0·47	0·53	0·57
" Iso.	700	0·81	0·51	...	0·60

2. *Blue-sensitive Emulsions.*

Plate.	Speed H. & D.	p (mean).	Values of q. Blue.	Violet.	Ultra-Violet.
Ilford Process	25	0·96	0·40	0·45	0·50
" Empress	100	0·94	0·54	0·54	0·57
Imperial Flashlight	400	0·90	0·58	0·61	0·57
Ilford Monarch	525	0·92	0·55	0·54	0·56
Barnet Stella	550	1·00	...	0·57	0·59
Ilford Zenith	650	0·90	0·54	0·61	0·66
Imperial Eclipse	650	0·92	0·59	0·61	0·62

considerable gain in accuracy by so doing, for as figs. 1 and 2 show, changes of density and development produce, in general, greater effects on q than on p, and these effects constitute important causes of error.

As in the case of panchromatic emulsions, it is found that when the density is low the coefficient d is very small, but that p rises with increase of density in most, though not in all cases, after the density passes the value unity. Fig. 2 shows a typical case; there is, however, some difference between different emulsions. As Table II shows, there may even be a fall of p as the density rises. Table III shows the relation between p and t for constant density, giving therefore values of the coefficient e in (1).

The conclusion that when the density is low p depends only on the exposure time, may be contrasted with the results of work at higher densities by Kron,* and more recently by Jones, Huse, and Hall.† These

* *Publ. des astroph. Obs. zu Potsdam*, **22**, 1913, Nr. 67.

† *Opt. Soc. Amer. Journ.*, **7**, 1923, p. 1079 ; **11**, 1925, p. 319 ; **12**, 1926, p. 321.

TABLE II.

Relation between p and D or log I, exposure time constant.

Plate.	Speed H. & D.	Log I/I_0	$\bar{1}\cdot4$.	0·0.	0·6.	1·2.
		D (approx.)	0·03.	0·30.	1·00.	1·80.
Ilford Process	25		0·94	0·96	0·95	0·95
" Empress	100		0·94	0·94	0·94	0·95
Imperial N.F. Ortho.	225		0·91	0·92	0·95	0·98
Ilford Screened Chrom.	270		0·91	0·90	0·89	0·87
Gevaert Filtered Ortho.	300		0·94	0·93	0·93	...
Ilford Auto-filter	400		0·95	0·95	0·96	1·02
Imperial Flashlight	400		0·91	0·91	0·93	...
" Monarch	525		0·91	0·92	0·94	0·98
" Eclipse	650		0·94	0·93	0·93	0·97
Ilford Zenith	650		0·87	0·88	0·89	0·94

(Values of p for exposure time 30 seconds.)

TABLE III.

Relation between p and t or log I, density constant.

Plate.	Speed H. & D.	Log I/I_0	$\bar{2}\cdot8$.	$\bar{1}\cdot4$.	0·0.	0·6.
		t (approx.)	600.	130.	30.	7 seconds.
Imperial N.F. Ortho.	225		0·86	0·89	0·93	0·97
Ilford Screened Chrom.	270		0·84	0·87	0·91	0·97
Imperial S.S. Ortho.	300		0·83	0·85	0·91	0·91
Gevaert Filtered Ortho.	300		0·87	0·90	0·92	0·96
Ilford Auto-filter	400		0·87	0·91	0·95	0·99
Imperial Flashlight	400		0·82	0·86	0·90	0·96
Ilford Zenith	650		0·81	0·84	0·90	0·96

(Values of p for density 0·3.)

latter workers approach the subject from a somewhat different point of view, and in effect imagine the density expressed as an explicit function of the common logs of the illumination I, and the exposure time t, thus

$$D = f(\log I, \log t).$$

They find that for the majority of the emulsions tested, the quantity

$$\gamma \equiv \frac{\partial D}{\partial(\log t)}$$

depends only on D (it is in fact nearly constant over a fixed range of density). If then we write

$$\frac{\partial D}{\partial(\log t)} = \frac{1}{\phi(D)}$$

and integrate, it follows that

$$\phi_1(D) = \theta(\log I) + \log t,$$

an equation used by Halm * as the basis of his photometric work. The value of p is given by

$$p = \frac{\dfrac{\partial D}{\partial(\log t)}}{\dfrac{\partial D}{\partial(\log I)}} = \frac{1}{\theta'(\log I)},$$

so that p depends only on the illumination. This is in agreement with the results of Kron's work. If it applied to low densities the values of p for the same emulsion and illumination contained in Tables II and III should agree within the limits of experimental error (about ·01). An examination of the tables from this point of view will show that, whatever happens at higher densities, at low densities p depends only on the exposure time. It is therefore possible to compare the values of p found for different effective wave-lengths without having to define the illuminations at which the comparison is to be made. The necessity for such a definition when the density is high has frequently been overlooked, and where p depends on the illumination no meaning can be attached without further explanation to such a statement as that p does not depend on the wave-length.

Values of p for constant exposure time are given in Table IV, the density being of the order 0·2 (Δ between $\bar{1}$ and 0), so that p is independent of the density or illumination. In all cases the dependence of p on the wave-length is of the same order as the experimental error. The results for panchromatic plates showed that p was affected by change of wave-length when the exposure time was long. No effect of this kind has been traced with isochromatic or blue-sensitive plates.

V. A Theory of the Deviations from the Reciprocity Law.

The connection between the illumination and duration of exposure of a photographic emulsion and the number of grains affected by the exposure to the extent of being made developable, is so fundamental a part of any theory of the photographic action that it is surprising to find so little

* *Monthly Notices R.A.S.*, 75, 1915, p. 159.

TABLE IV.

Values of p for low density and for various times and wave-lengths.

1. *Isochromatic and Screened Emulsions.*

Plate.	Exposure time (mean).	Values of p. Wratten filter 22. Effective wave-length 5700.	77. 5400.	74. 5300.	36. 4100.	18. 3650.
	seconds					
Ilford Screened Chromatic .	7	...	...	0·992	1·015	0·993
" " .	40	...	...	0·912	0·920	0·890
" " .	180	...	...	0·850	0·871	0·853
" " .	800	...	...	0·840	0·835	0·820
Wellington Anti-screen . .	30	0·783	0·782	...	0·782	0·791
Ilford Auto-filter . . .	30	0·946	0·946	...	0·938	0·953
Gevaert Filtered Ortho. . .	30	0·903	0·898	...	0·905	0·923

2. *Blue-sensitive Emulsions.*

Plate.	Exposure time (mean).	Values of p. Wratten filter 75. Effective wave-length 4800.	36. 4100.	18. 3650.
	seconds			
Ilford Zenith	8	0·939	0·965	0·935
" "	16	0·901	0·903	0·891
" "	42	0·899	0·892	0·916
" "	180	0·845	0·835	0·822
" "	800	0·797	0·806	0·806
Imperial Eclipse	20	0·911	0·909	0·906
Ilford Empress	35	0·930*	0·934	0·931
" Process	35	0·961*	0·963	0·951

importance attached in any current theory to the deviations from the reciprocity law; the more so since in many theories the number of grains affected is supposed to bear an exact numerical relation (sometimes indeed to be equal) to the number of molecules of the hypothetical substance termed the latent image. No such far-reaching assumption is necessary for the study of the deviations from the reciprocity law; it need only be assumed that the deposit formed by full development is a measure of the action which has taken place. In the commercial emulsion the diversity

* Ilford Beta Filter.

in size of the grains and the screening effect of those in the front layers on those lying behind might invalidate even this assumption; so, in the first place, the results of experiments on plates with a single layer of uniform grains will be considered.

In a series of experiments of this nature Dr R. E. Slade and Mr G. I. Higson concluded * that their results could be expressed by the empirical formula

$$\log_e \frac{a}{a-x} = k_0 \mathrm{I}(1-e^{-k_3\mathrm{I}})\{t - k_5(1-e^{-k_6 t})\} \quad . \quad . \quad . \quad (1)$$

where a is the number of grains in unit area of the emulsion, x the number affected by light, I the illumination, and t the duration of the exposure, and k_0, k_3, k_5, k_6, constants. This formula will now be considered further.

If the illumination I is very great and the exposure time t small, the expression (1) reduces to

$$\log_e \frac{a}{a-x} = k_0 \mathrm{I}t(1 + k_5 k_6) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

that is to say, the reciprocity law is obeyed. If, on the other hand, I is very small and t very great, it reduces to

$$\log_e \frac{a}{a-x} = k_0 k_3 \mathrm{I}^2 (t - k_5) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

k_5 being eventually small in comparison with t. It appears then that when the exposure time is short the effect of the light depends on the value of $\mathrm{I}t$; but that when it is long the effect depends on the value of $\mathrm{I}^2 t$.

If, now, quanta are treated as molecules obeying the law of mass action, their "active mass" being proportional to the illumination, an action of the type

$$\mathrm{A} + \mathrm{Q} \longrightarrow \mathrm{AQ},$$

A representing an active molecule and Q a quantum, or any series of such actions,

$$\begin{gathered} \mathrm{A} + \mathrm{Q} \longrightarrow \mathrm{AQ} \\ \mathrm{AQ} + \mathrm{Q} \longrightarrow \mathrm{AQ}_2 \\ \cdots\cdots \end{gathered}$$

proceeds in accordance with the reciprocity law. On the other hand, a "trimolecular" action of the type

$$\mathrm{A} + 2\mathrm{Q} \longrightarrow \mathrm{AQ}_2$$

gives a result depending on the value of $\mathrm{I}^2 t$.

It must be supposed, then, that *two* quanta are concerned in the photographic action, and that the two must be absorbed within a short interval

* *Proc. Roy. Soc. Lond.*, **98**, 1920, p. 154.

of time, giving the effect of two distinct absorptions when the exposure is short, and of a simultaneous absorption when the exposure is long. This indicates that in the absence of a second absorption the effect of the first quantum is transitory, the actions being of the type

$$\mathrm{A} + \mathrm{Q} \rightleftharpoons \mathrm{AQ}$$
$$\mathrm{AQ} + \mathrm{Q} \longrightarrow \mathrm{AQ}_2 .$$

If the resulting expression for the course of the light action is to be checked by the experimental results, which even in the case of Slade and Higson's experiments are of none too great accuracy for the purpose, further assumptions must be introduced to cut down the number of arbitrary constants. The two quanta, it appears, must be taken up by different components of the absorber, for otherwise it would be expected that a single quantum, of frequency more than double that necessary to cause any action, would be sufficient to cause the whole effect. This would lead to a discontinuity in the ultra-violet, and no such discontinuity has been observed. Next, the maximum wave-lengths effective in the two stages must be close to the maximum for any photographic action. To test this point, crossed spectra of the mercury arc were exposed simultaneously on a plate. The resulting negative showed no appreciable increase of density where the green line of one spectrum crossed the violet line of the other, so that the maximum effective wave-length is in both cases less than 5461 Å. This similarity between the components leads to the further supposition that either may absorb the first quantum, and also that the probability of either absorbing a quantum is the same. These tentative assumptions applied to the equations

$$\left.\begin{array}{l} \mathrm{A} + \mathrm{Q} \rightleftharpoons \mathrm{B} \\ \mathrm{B} + \mathrm{Q} \longrightarrow \mathrm{C} \end{array}\right\} \qquad (4)$$

lead to the following dynamical equations:—

$$\left.\begin{array}{l} \dot{x}_1 = -2h\mathrm{I}x_1 + kx_2 \\ \dot{x}_2 = 2h\mathrm{I}x_1 - (h\mathrm{I} + k)x_2 \\ \dot{y} = h\mathrm{I}x_2 \end{array}\right\} \qquad (5)$$

where x_1, x_2, y, are the numbers of double absorbers in the stages A (original active stage); B (transition stage); and C (latent image); and h and k are constants. These give, on integration,

$$y = n\left\{1 - \frac{re^{-st} - se^{-rt}}{r - s}\right\} \qquad (6)$$

n being the total number of double absorbers, and r and s the roots of the equation

$$z^2 - (3h\mathrm{I} + k)z + 2h^2\mathrm{I}^2 = 0 \qquad (7)$$

In the special cases of very intense or very weak illumination the number of arbitrary constants may be reduced from two to one. In the former case (I very great) the roots of (7) are approximately $h\mathrm{I}$ and $2h\mathrm{I}$, giving on substitution in (6)

$$y = n(1 + e^{-2h\mathrm{I}t} - 2e^{-h\mathrm{I}t}) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

On the other hand, when I is small and t great, the roots of (7) are approximately k and $\frac{2h^2\mathrm{I}^2}{k}$, and (6) becomes

$$y = n(1 - e^{-\frac{2h^2\mathrm{I}^2 t}{k}}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

These relations between y, the number of "molecules of latent image," and n, the number of double absorbers, cannot be compared with experimental results dealing with numbers of *grains* without further assumptions. Under suitable development conditions a grain is either completely blackened or else totally unaffected; the conclusion is drawn that a single molecule of latent image is sufficient to render a grain developable. There remains the question of the number of double absorbers in a grain. The observation that there are few, if any, inactive grains may be accounted for by supposing, either that each grain contains a large number of absorbers, which are distributed at random among them, or else that a grain necessarily contains at least one absorber, as would be the case if the absorbers form part of nuclei on which the grains have crystallised. The formulæ will then be compared with the experimental results on the alternative suppositions (1) that a grain contains a large number of absorbers, and (2) that it contains only one double absorber.

As before, let a be the number of grains, x the number affected by light, n the number of double absorbers, and y the number of latent image molecules. On the assumption of one double absorber in each grain, it follows that $n = a$, $x = y$, equations (6) to (9) retaining the same form. If, however, n is of a higher order than a, the probability that a given grain contains at least one of the y molecules is

$$1 - \frac{(a-1)^y}{a^y}$$

so that

$$\frac{x}{a} = 1 - \frac{(a-1)^y}{a^y}$$

$$= 1 - e^{-\frac{y}{a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Now n is by hypothesis of a higher order than a, while (10) shows y and a to be of the same order. The values of $\frac{y}{n}$ given by (8) and (9) are then small, and may be reduced to their first terms, namely, those in I^2, giving respectively when substituted in (10), for strong illuminations

$$\log_e \frac{a}{a-x} = \frac{nh^2 I^2 t^2}{a} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and for weak illuminations

$$\log_e \frac{a}{a-x} = \frac{2nh^2 I^2 t}{ka} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

It will be observed that (12) is of the same form as (9), both being identical with the empirical equation (3), if in that equation the constant k_3 is negligible. The possibility of deciding in this manner between the two assumptions regarding the number of absorbers in a grain depends then on the difference between the equations (8) and (11).

The particular experiments selected from the paper quoted above for comparison with (8), (11), and (9) or (12) are Nos. 7 and 16 for strong light and Nos. 10 and 15 for weak light.* The reason for the rejection of the experiments with intense monochromatic light will be given later. This reason does not apply to the case of experiment 15 with weak monochromatic light, which has been included in default of corresponding data for white light. Each of the columns of calculated results depends on only one arbitrary constant, the constants being chosen independently in each case to give the best agreement. The results show clearly that the assumption of one double absorber in each grain gives remarkably good agreement with the observed values, and that the assumption of a large number in each grain is untenable.

1. *Short Exposures, formulæ* (8) *and* (11).

(A) Exposure time constant (less than one second).

I (in candle-metres).	Percentage of grains changed.		
	Calculated from (8).	Calculated from (11).	Observed.
775	100·0	100·0	100 app.
378	99·9	100·0	100 „
185	95·1	99·9	94·3
90·4	69·9	80·5	71·9
63·1	51·5	54·9	52·4
44·2	34·4	32·3	31·2
21·6	12·3	8·9	12·0
10·5	3·6	2·2	4·6
5·15	0·9	0·5	0·4
2·52	0·2	0·1	Nil.

* R. E. Slade and G. I. Higson, *loc. cit.*, pp. 163, 167, 168.

(B) Illumination constant (44·5 candlemetres).

t (seconds).	Percentage of grains changed.		
	Calculated from (8).	Calculated from (11).	Observed.
10	94·1	99·2	91·5
5	68·3	69·9	64·6
2·5	34·0	25·9	36·2
1·25	12·6	7·2	15·8

2. *Long Exposures, formula* (9) *or* (12).

(A) Exposure time constant (ten minutes).

I (in candle-metres).	Percentage of grains changed.	
	Calculated.	Observed.
1·89	100·0	100 app.
0·927	100·0	100 „
0·453	96·3	98 „
0·316	79·8	80·0
0·221	54·2	53·0
0·108	17·0	15·0
0·0528	4·4	5·3
0·0258	1·1	1·7
0·0126	0·3	0·8

(B) Illumination constant (monochromatic, units arbitrary).

t (seconds).	Percentage of grains changed.	
	Calculated.	Observed.
600	66·0	64·5
500	59·3	60·0
400	51·3	57·4
300	41·7	42·1
200	30·2	32·5
100	16·5	18·0

Experiments of this kind give values for the quantities h and k; results for short exposures giving h, and those for long exposures $\frac{h^2}{k}$. In this series of experiments, however, conditions are not altogether favourable, for the different experimental results are not quite consistent one with another; but where the values of the illuminations are given in comparable units the deduced value of k is, for white light, about 0·01 sec^{-1}; for monochromatic light, about 1 sec^{-1}. This difference between the behaviour of white and monochromatic light is ascribed by Messrs Slade and Higson to some fundamental distinction between the action of homogeneous light and that of white light; but this view is difficult to reconcile with the absence of peculiarities from photographed spectra, and the phenomenon is probably due to the intermittent nature of the discharge in the mercury lamp used.* In such cases it is found that when the period of the intermittent light is short in comparison with the total exposure, the light acts as though it were continuous; † but as the exposure is shortened the effective illumination must approach and finally reach the maximum illumination in the flash, while the total exposure time may be several times as long as the actual exposure to light. The phenomena observed with intermittent light, it may be observed, are completely in accordance with the theory given in this paper, though the lack of suitable experimental data for comparison makes a quantitative treatment of little interest.

In the case of a commercial emulsion, the connection between the density of the deposit and the number of blackened grains it contains depends on so many factors ‡ as to offer little assistance in verifying the equation (6). Matters are much simpler when the density is very low, for in that case both theory and experiment show the deposit to be practically confined to the front layer of grains. It follows that the light absorbed from a parallel beam by a weak deposit is proportional to the projected area, and therefore approximately to the number, of the developed grains. Again, when the deposit is weak, (6) may be put in a simpler form by taking the product It, denoted by E, as small. The equation (6) may be written in the form

$$y = n\left\{1 - \frac{rt \,.\, e^{-st} - st \,.\, e^{-rt}}{rt - st}\right\}$$

where rt and st are the roots of the equation

$$z^2 - (3h\mathrm{E} + kt) + 2h^2\mathrm{E}^2 = 0.$$

If $h\mathrm{E}$ is so small as to be small in comparison with kt, the roots are kt and $\frac{2h^2\mathrm{E}^2}{kt}$. Substituting and retaining terms in E^2, the equation (6) becomes

$$y = \frac{2h^2\mathrm{E}^2 n}{k^2t^2}(e^{-kt} + kt - 1) \quad . \quad . \quad . \quad . \quad . \quad (13)$$

* *Cf.* Arons, *Ann. d. Physik*, **58**, 1896, p. 91.

† Weber, *Ann. d. Physik.*, **45**, 1914, p. 801.

‡ See P. G. Nutting, *Phil. Mag.*, **26**, 1913, p. 423.

This result applies even when kt is of the same order as hE, for the roots are then both small quantities of this order and (6) may be expanded, the result as far as the second order term being, as in (13) when kt is small, the quantity $2h^2E^2n$.

Equation (13) may be used to give theoretical values of the quantities p and q used in the earlier parts of this paper and the preceding paper (see p. 01). If the transparency of the deposit to parallel light is denoted by T, the light absorbed is $1-T$, which, as stated above, is proportional to y when the density is low. Since, in that case T is nearly unity, we may write $\frac{1-T}{T}$ for $1-T$, thus—

$$\frac{1-T}{T} \propto \frac{2h^2I^2}{k^2}(e^{-kt}+kt-1) \quad . \quad . \quad . \quad . \quad (14)$$

which gives p and q at once on differentiation, the results being

$$\left.\begin{aligned} p &= \frac{1-e^{-kt}}{2(e^{-kt}+kt-1)} \\ q &= \tfrac{1}{2}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The tendency for q to approach the value 0·5 * as the density is diminished has been remarked both in this and in the previous paper. The fact that

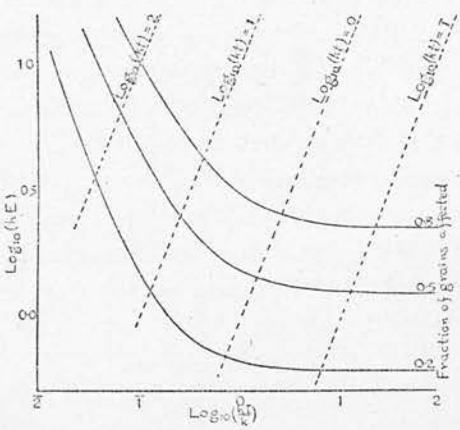

FIG. 7A.—Theoretical curves of constant density, process emulsion.

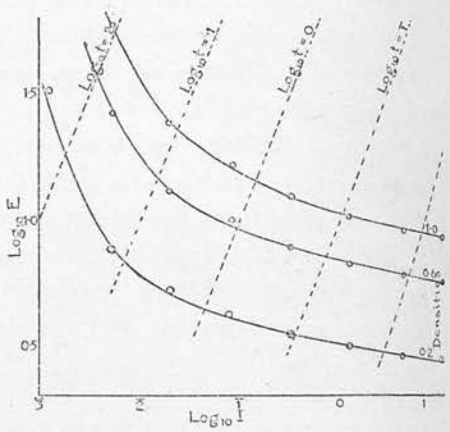

FIG.—7B. Observed curves of constant density, process emulsion (Jones, Huse, and Hall).

when the density is low p depends on the exposure time and not on the illumination (see above, p. 04) is in agreement with (15), but the variation of p with t given by that formula and shown in fig. 8, C., is far from being exact, even in the case of process plates. Fig. 7 shows a comparison with

* It may be noted here that the same reasoning applied in the case of action by a single quantum gives the value unity for q at low densities.

the results given by L. A. Jones, E. Huse, and V. C. Hall * for a process emulsion. Since their results are given in the form of values of log E and log I giving particular densities, corresponding results for particular percentages of grains affected, calculated from (6), are given.† It will be noted that the constants in (6) serve only to fix the zeros of the abscissæ and ordinates, the shapes and orientations of the curves being determinate. A further disagreement with observation occurs in the case of short exposures to intense light, when p is found to rise above unity, which, according to (6), would be its maximum value. This effect is almost certainly to be associated with the destruction of the latent image known as "reversal," the theory of which may therefore be examined.

There are two theories purporting to account for reversal, the one attributing the disappearance of the latent image to the action of bromine liberated by light from the bromide of silver grains,‡ the other to the direct action of light on the latent image.§ Either theory could be modified to account for values of p greater than unity, the former by supposing the action of light on silver bromide to take place in two stages, in the manner supposed above for the formation of the latent image, but with a much shorter-lived transition product; the latter by supposing the decomposition of the latent image, like its formation, to occur in two stages, the transition product being again short-lived.

The direct decomposition of silver bromide by light has been shown by P. P. Koch & H. Vogler || to result in silver as one product; that it takes place in two stages through a luminescent intermediate stage is very probable, since it is found that silver bromide is luminescent. Experiment shows that the luminescence does not persist for more than a fraction of a second after the withdrawal of the illumination, in agreement with the idea that the transition stage is short-lived. On the other hand, the efficacy of red light in reversal ¶ is not easily accounted for on the bromine theory, since silver bromide does not absorb appreciably in the red, whereas the latent image itself might be black. Unfortunately, there is little experimental data by which the results of the light action theory, which are easily computed, might be checked. Assuming, however, that the reversal action involves

* *Opt. Soc. Amer. Journ.*, 12, 1926, p. 329.

† There is some doubt as to the unit employed for t in the results given by Jones, Huse, and Hall. If this were 1 second, the value of p corresponding to a 30 seconds' exposure would be about 0·6. This is much lower than any value found here. The Ilford process plates tried gave the value 0·95.

‡ S. E. Sheppard, "Photography as a Scientific Implement" (1923), p. 179.

§ C. W. Piper, *Brit. Journ. Phot.*, 55, 1908, p. 195.

|| *Ann. der Physik*, 77, 1925, p. 495.

¶ *Cf.* H. S. Allen, *Photo Electricity*, 1st ed., p. 206.

the absorption of *four* quanta and the addition to the equations (4) of another pair of similar equations, thus

$$\left.\begin{aligned} A + Q &\rightleftharpoons B \\ B + Q &\longrightarrow C \\ C + Q &\rightleftharpoons D \\ D + Q &\longrightarrow E \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

a solution on the same lines as before shows, as might be expected, that in the expression for the number of developable grains (those grains, that is, in the stages C and D) the terms involving I^2 are unaffected by the additional equations, which introduce terms involving the fourth and higher powers of the illumination. The prominence of reversal in extremely short exposures to high intensities is thus well accounted for by the theory of direct light action on the latent image, but the expression (15) remains unaffected.

It seems, then, that though values of p exceeding unity are readily accounted for as the result of the reversal action, the divergence of (15) from the observed results for process emulsions must be ascribed to some other cause, the action being more complicated than has been supposed. Assume, therefore, that the absorbers which make up the double absorber are not exactly similar, though still resembling each other in so far that either may take up the first quantum. This replaces the equations (4) by the equations

$$\left.\begin{aligned} A + Q &\rightleftharpoons B_1 \\ A + Q &\rightleftharpoons B_2 \\ B_1 + Q &\longrightarrow C \\ B_2 + Q &\longrightarrow C \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

there being now two different transition stages according to the absorber first affected. Suppressing superfluous constants, the most general form of the corresponding dynamical equations is

$$\begin{aligned} \dot{x}_1 &= -(I + J)x_1 + kx_2 + lx_3 \\ \dot{x}_2 &= Ix_1 - (K + k)x_2 \\ \dot{x}_3 &= Jx_1 - (L + l)x_3 \\ \dot{y} &= Kx_2 + Lx_3, \end{aligned}$$

reversal being neglected. The solution is

$$\frac{y}{a} = \frac{\mu\nu - KI - LJ}{(\nu - \lambda)(\lambda - \mu)} e^{-\lambda t} + \frac{\nu\lambda - KI - LJ}{(\lambda - \mu)(\mu - \nu)} e^{-\mu t} + \frac{\lambda\mu - KI - LJ}{(\mu - \nu)(\nu - \lambda)} e^{-\nu t} + 1$$

where λ, μ, ν are the roots of the cubic

$$z(z - I - K - k)(z - J - L - l) + LJ(z - K - k) + KI(z - L - l) = 0.$$

The effective illuminations I, J, K, L, all being supposed small quantities of the first order, these roots are

$$k,\ l,\ \frac{\mathrm{K}\mathrm{I}l+\mathrm{L}\mathrm{J}k}{kl}.$$

Proceeding, as in forming (13), by supposing the product of time and illumination to be small and of the second order, we have

$$\frac{y}{a}=\mathrm{KI}\,.\,\frac{e^{-kt}-1+kt}{k^2}+\mathrm{LJ}\,.\,\frac{e^{-lt}-1+lt}{l^2} \qquad . \quad . \quad . \quad (18)$$

or if, as before, the illuminations (or probabilities of quantum absorption) are equal throughout,

$$\frac{y}{a}=\mathrm{I}^2\frac{e^{-kt}-1+kt}{k^2}+\mathrm{I}^2\frac{e^{-lt}-1+lt}{l^2}$$

whence

$$p=\frac{\dfrac{1-e^{-kt}}{kt}+\dfrac{1-e^{-lt}}{lt}}{2\,.\,\dfrac{e^{-kt}-1+kt}{k^2t^2}+2\dfrac{e^{-lt}-1+lt}{l^2t^2}}.$$

The application of this formula gives good results in the case of process plates, as shown in fig. 8, where the curve E is deduced from the observa-

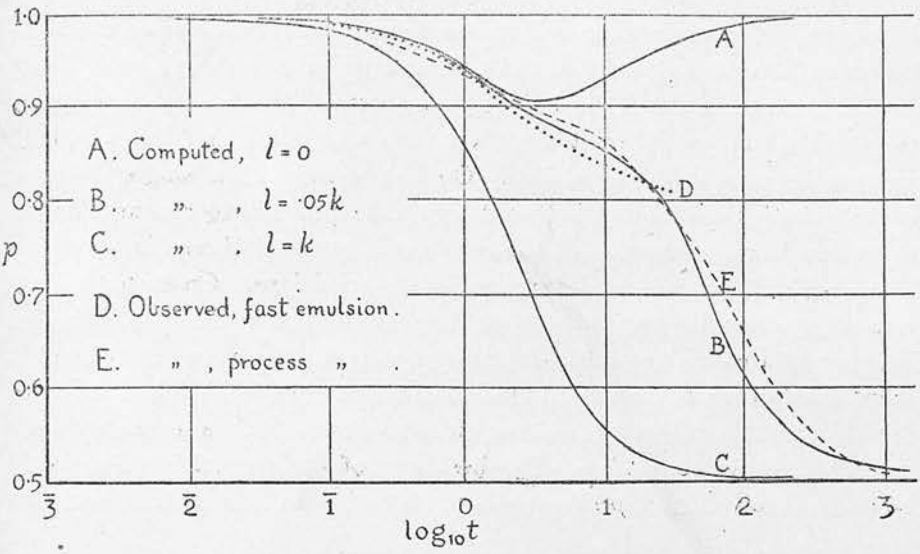

FIG. 8.—Relation between p and $\log t$ for low densities and moderate or low illuminations.

tions of Jones, Huse, and Hall, given in fig. 7 (the unit of time being arbitrary).

There is another way in which, not only these observations, but also those for the fast emulsion, can be accounted for. This is to suppose that instead of the two components of an absorber being different, there are two or more different absorbers present, having similar components. (It will be noticed that mere diversity of grain size will have no effect on the value of p given by (14) unless at least two varieties of absorbers are present.) The resulting formula is similar to (18). It appears that the agreement found previously in the case of formulæ (8) and (9) between theory and experiment is practically unaffected by these modifications. It is not certain, however, that the low density results for fast emulsions will differ greatly from those for process emulsions. The writer's results do not cover the very long exposures, while the difference found by Jones, Huse, and Hall may be attributed to the much greater maximum density of the process emulsion, so that a fixed density corresponds to a much smaller proportion of the grains in its case than in the case of the fast emulsion.

The only remaining effect which persists at the lowest densities is the effect of development on p. On the view that two different absorbers are present this would be ascribed to the grains made active by the one kind developing faster than those made active by the other.

The significance of these results in the theory of the latent image is left open. The main assumptions could be incorporated without much difficulty into most theories of the process, and for this reason an attempt has been made to avoid the language of specific theories. It is tempting to associate the two different absorbers with the iodine and bromine, both present in commercial emulsions. On the view that a single absorber contains both bromine and iodine it would be possible to account for the extension towards the red of the sensitiveness of iodo-bromide emulsions compared with pure bromide or pure iodide emulsions. On such a theory, however, p should depend greatly on the wave-length in the spectral region between the commencement of photographic sensitiveness and the commencement of sensitiveness of an iodide emulsion. Filtered light is unsuited for observations over a spectral region so narrow as this, and the evidence is not altogether conclusive. The best way of deciding this point would be to find the values of p for pure bromide and pure iodide emulsions.

(Issued separately , .)

ON THE VALIDITY OF TALBOT'S LAW FOR THE PHOTOGRAPHIC PLATE

BY

E. A. BAKER, B.Sc.

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Part I.

(APPENDIX 2.)

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ON THE VALIDITY OF TALBOT'S LAW FOR THE PHOTOGRAPHIC PLATE

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ABSTRACT. The action of intermittent light on a photographic plate may be represented approximately by a law similar to that of Schwarzschild for continuous light, viz., $f(\mathrm{D}) = \mathrm{I}t^{\pi}$, where D is the density, I the illumination, and t the duration of actual exposure to light. If the sector is revolved once only, π is equal to Schwarzschild's index p; but if Talbot's law applies, π will become unity for sufficiently high sector speeds. The values of π are given for the special case of an 180° sector and total exposures of the order of one minute, under circumstances for which the range of p was ·71 to 1·06; of wave-length, ·61μ to ·36μ; and of plate speed, 5 to 700 H. & D. The results show that the deviation of π from unity for speeds giving over 60 flashes per second is less than 1 per cent.; but that for 800 flashes per second the deviation from unity may still be appreciable.

In visual and photo-electric photometry the rotating sector provides perhaps the most trustworthy method of reducing the intensity of a beam of light in a known ratio; but in photographic photometry its use has been restricted, for it is known that when the speed of the sector is very low a law similar to Talbot's law in visual photometry is not valid for most photographic plates. When the speed is reduced to its minimum—that giving one revolution during the exposure—the behaviour of the plate may be roughly expressed in the form due to Schwarzschild, viz.—

$$f(\mathrm{D}) = It^{p},$$

where D is the density of the deposit, I the illumination and t the exposure time. When the speed of rotation is more rapid the plate may be supposed to obey a similar law, viz.—

$$f(\mathrm{D}) = \mathrm{I}t^{\pi},$$

an assumption which is the basis of a well-known method of spectrophotometry in the ultra-violet.[1] It has been concluded, in particular by Weber [2] and by workers at the Bureau of Standards,[3] that even for moderate sector speeds the exponent π does not differ measurably from unity, though it is scarcely possible to verify this for all the combinations of sector angle, sector speed, illumination, total exposure time, and other conditions which might

[1] See catalogues of Messrs. Adam Hilger, Ltd.

[2] *Ann. d. Phys.*, 45 (1914), 801.

[3] *Bureau of Standards Scientific Papers*, No. 440 (1922). Other references are given in this paper.

well affect its value. Little attention appears to have been paid to the variable which might be expected to affect π more than any other—that is, the value of p, which varies over a wide range even for plates nominally identical. Again, the light source used by Weber in most of his work was the mercury arc, which by reason of its discontinuity [4] might be expected to introduce a factor absent when continuous light is employed.

The present determinations of the value of π were made in circumstances chosen to give a wide range of values for p, which was determined under practically identical circumstances. The experimental method used could deal with only one variety of sector—that giving equal intervals of light and darkness—and considerations of accuracy restricted the range of exposure times to from 40 to 60 seconds, as being large enough for accurate work with a simple hand shutter, and yet small enough to render possible a considerable number of repetitions of the experiments.

The results are given in the table on opposite page.

It appears that even for so low a speed as that giving 750 flashes per minute the deviation of π from unity is less than 2 per cent., and that for the moderate speed of 3000 flashes per minute all trace of the influence of p is lost. The fact that at the two highest sector speeds the value of π is distinctly greater than unity would naturally be ascribed to the systematic errors of the experimental methods, and it is from this point of view that they will be discussed.

The apparatus used is shown in section in Fig. 1. The light source was an ordinary 100 watt gas-filled lamp A run at its marked power consumption. The current was controlled to an accuracy of 1 in 1000, giving an approximate accuracy of at least 1 in 200 in the light flux. The filament of the lamp was approximately in one plane, and a lens B in this plane gave a very fairly straight and narrow image of the filament. The image fell along a radius of the sector C, so that there was no appreciable interval between darkness and full illumination. The light of the image entered the camera D through a narrow slot E and a filter F, and could reach the photographic plate P only by way of two apertures GG, each provided with a prism to deflect the light towards the plate, a set of Waterhouse stops and a ground glass window. The light passing through either aperture could be occulted by a piece of blackened brass H fitting the slots for the stops. No effort was made to secure perfect equality of the light beams passing through the two apertures—the only assumption made is that when both are open, the illumination of the plate is the sum of the illuminations obtained by occulting either aperture in turn. The image given by exposing through both apertures with the sector running was then compared with those given by the single apertures in turn when the sector was stationary with the light passing freely through one of its apertures. The method of working and the allowance for inequality of the beams has been described in another application of the same photometric method.[5] The sector

[4] Cf. M. Leblanc fils, *L'arc éléctrique* (Blanchard, Paris, 1922), 79.
[5] *Proc. Roy. Soc. Edinburgh*, 45 (1925), 166.

Emulsion.	Speed (H. & D.).	Light and filter.	Exposure time.	Density.	Values of π for a 180° sector rotating at 1 r.p.m. (p).	750 r.p.m.	3000 r.p.m.	45,000 r.p.m.	Camera.
Wellington iso. . .	700	Ultra-violet (Wratten 18) .	40 secs.	·5	·82	—	1·009	—	B
,, ,, . .	,,	White (no filter) . .	,,	1·0	·84	—	—	1·009	A
,, ,, . .	,,	Minus ultra-violet (Ilford Q)	,,	·75	·83	—	—	1·010	A
,, ,, . .	,,	Yellow (Wratten K2) .	,,	·20	·79	—	—	1·003	A
,, anti-screen	450	White (no filter) . .	,,	1·1	·87	—	—	1·021	A
,, ,, .	,,	Minus ultra-violet (Ilford Q)	,,	·9	·84	—	—	1·007	A
,, ,, .	,,	Yellow (Wratten K2) .	,,	·25	·80	—	—	·997	A
Ilford auto-filter . .	400	White (no filter) . .	45 secs.	1·7	1·06	—	—	1·005	A
,, ,, . .	,,	Minus ultra-violet (Ilford Q)	,,	1·5	1·06	—	—	1·010	A
,, ,, . .	,,	Yellow (Wratten K2) .	,,	·45	·97	—	—	1·025	A
,, rapid chromatic	,,	Ultra-violet (Wratten 18) .	40 secs.	·7	·89	—	1·010	1·001	B
Imperial panchromatic	350	Ultra-violet (Wratten 18) .	60 secs.	·07	·88	·976	1·018	1·001	B
,, ,, .	,,	Violet (Wratten θ) . .	,,	·40	·86	·983	1·006	1·008	B
,, ,, .	,,	Green (Wratten ϵ) . .	,,	·08	·89	1·000	1·006	·992	B
,, ,, .	,,	Orange (Wratten γ) . .	,,	·25	·85	·988	1·004	1·003	B
,, ,, .	,,	White (no filter) . .	40 secs.	1·3	·72	—	—	1·006	A
,, ,, .	,,	Yellow (Wratten K2) .	,,	1·0	·71	—	—	·998	A
,, rapid lantern .	5	White (no filter) . .	,,	1·7	1·01	1·004	1·000	1·013	B

was a brass disc with truly parallel faces to give good balance at high speeds, and had six arms and six apertures between, all adjusted as closely as possible to 30°. It was mounted on an axle running in ball bearings and driven by a belt from a motor. A circle was marked on the face of the sector at the position of the image by rotating it in its bearings in contact with a scriber, and the chords cut by the radial edges from this circle were measured on a machine. The transparency of the sector, assuming Talbot's law, was given by these measurements as $50{\cdot}17 \pm {\cdot}02$ per cent.

The plates were developed with the developer (metol-quinol or pyro-soda) recommended by the makers, and the time of development chosen to give practically full development without appreciable fog. The density measurements were made with a photo-electric photometer reading to ·002. Each reading is relative to two readings on the neighbouring clear plate. Each value given in the table is the mean of four determinations, and therefore depends on twelve exposures or thirty-six photometer readings.

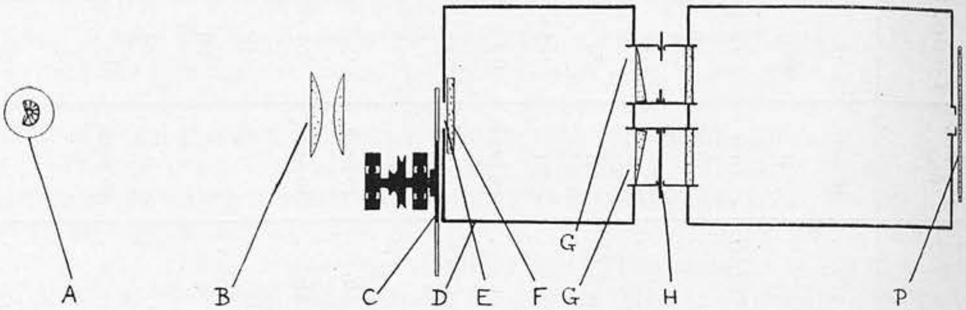

FIG. 1.—Section of apparatus.

The mean of the first series of observations—those taken with camera A (last column of table) showed a definite deviation of π from unity by $\cdot 008 \pm \cdot 002$. It was estimated that any error arising from scattered light or reflections was below ·002, and the reason for the deviation was looked for in the imperfect superposition of the two patches of light on the plate with both apertures open. Owing to the Eberhard effect and to halation, the density across a light patch is not constant. The Eberhard effect alone produces a minimum of density in the centre of the patch, while the effect of halation is much more complicated. (The plates were not backed.) Both effects should be negligible at low densities. A new camera (camera B) was made in which the separation between the two apertures, which in camera A was one-eighth of the distance between apertures and plate, was reduced to one-twentieth of that distance. The determinations made with this camera at the highest speed of rotation give a mean value of π of $1{\cdot}003 \pm {\cdot}002$. At the speed giving 3000 flashes per minute, however, this is increased to $1{\cdot}008 \pm {\cdot}002$. The difference cannot be ascribed to systematic error, for the moving of the belt to a different pulley could scarcely affect the system to this extent. The error may be accidental; but it is probable that π rises above unity as the speed is increased, and with further increase of speed falls and approaches unity.

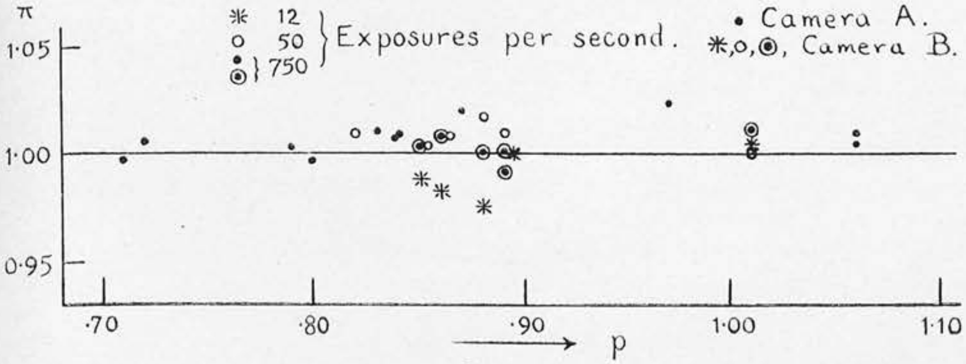

Fig. 2

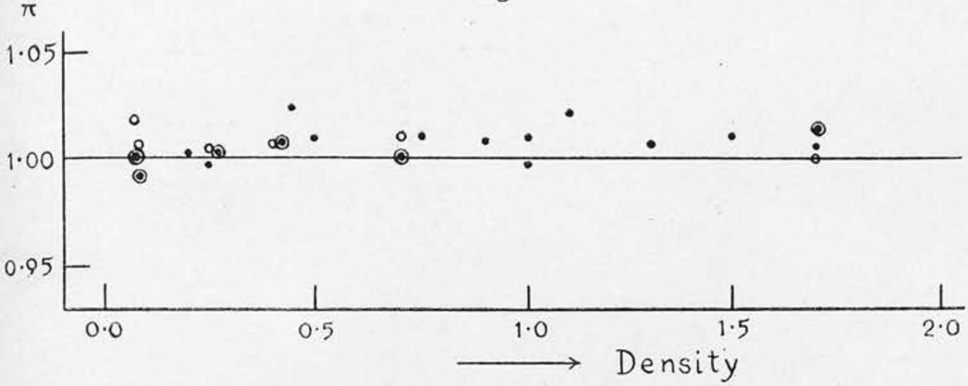

Fig. 3.

The observations are shown plotted against p and against D in Figs. 2 and 3. The values of p found for one batch of Imperial panchromatic plates were exceptionally low, and unfortunately depend on only one plate, the last of that batch. The value 1·06 for the batch of Ilford auto-filter plates was definitely verified by other plates.

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